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Myopic Oligopoly Pricing*

Iwan Bos[†] Marco A. Marini[‡] Riccardo D. Saulle[§]

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Abstract

This paper examines capacity-constrained oligopoly pricing with sellers who seek myopic improvements. We employ the Myopic Stable Set stability concept and establish the existence of a unique pure-strategy price solution for any given level of capacity. This solution is shown to coincide with the set of pure-strategy Nash equilibria when capacities are large or small. For an intermediate range of capacities, it predicts a price interval that includes the mixed-strategy support. This stability concept thus encompasses all Nash equilibria and offers a pure-strategy solution when there is none in Nash terms. In particular, it provides a behavioral rationale for different types of pricing dynamics, including real-world economic phenomena such as Edgeworth-like price cycles, price dispersion and supply shortages.

Keywords: *Behavioral IO, Bounded Rationality, Capacity Constraints, Oligopoly Pricing, Myopic Stable Set.*

JEL Codes: *C72, D43, L13.*

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Contents

1 Introduction

A common assumption in the literature on oligopoly pricing is that firms aim to maximize their profits.[1] In game-theoretic terms, players are presumed to pick *best-responses* to each other's choices. Although it may be reasonable to assume such maximizing behavior, there are compelling arguments for why sellers sometimes make suboptimal decisions. They simply need not be fully rational, for instance, or make mistakes. Also, they might lack the information to identify their most preferred alternative. For example, a firm may not be able to precisely determine its profit-maximizing price *ex ante* and regret its decision *ex post*, *i.e.*, after it has observed the actual choices of its competitors.

With this in mind, this paper offers a novel perspective on oligopoly pricing by postulating that sellers are *myopic* and simply aim to improve upon their current situation. Specifically, we analyze a model of price competition with capacity constraints under the assumption that firms choose better- rather than best-responses. This in particular means that they can, but may not, behave like the neoclassical profit-maximizing firm.

Under this assumption, we employ the concept of *Myopic Stable Set* (MSS), which was recently introduced by Demuynck, Herings, Saulle and Seel (2019a). A set of price profiles is *myopically stable* when it satisfies three conditions: deterrence of external deviations, asymptotic external stability and minimality. The 'deterrence of external deviations' requirement holds when none of the sellers gains by moving from a price profile in the MSS to a price profile outside the MSS. 'Asymptotic external stability' ensures that from any price profile outside the set it is possible to get arbitrarily close to a price profile inside the MSS through a sequence of domination steps. Finally, 'minimality' holds when the MSS is minimal with respect to set inclusion.

We establish the existence of a unique MSS for any given level of capacities. In terms of characterization, we show that if the set of pure-strategy Nash equilibria is nonempty, then it coincides with the MSS. A corollary to this is that the MSS reduces to the pure-strategy solutions that exist for sufficiently large or small production capacities. If capacities are in an intermediate range, then typically there is no pure-strategy Nash equilibrium. In these cases, there is a mixed-strategy Nash equilibrium, the support of which is shown to be contained in the MSS. The MSS therefore also permits price dispersion, but the range of possible 'sales' is wider than in a mixed-strategy Nash equilibrium. Taken together, we then find that the behavioral assumption of sellers simply seeking myopic improvements does not *qualitatively* affect existing (Nash) price predictions.

The perspective taken in this paper has, however, several advantages over the standard Nash approach to oligopoly pricing and to capacity-constrained price competition in particular. The MSS solution concept, for example, rests on a less-stringent behavioral assumption since sellers are supposed to behave myopically and choose better- rather than best-responses. Yet, they never-

[1] An in-depth discussion of classical models of oligopoly pricing is provided by Vives (1999).

theless charge precisely the same prices as they would have in a pure-strategy Nash equilibrium. Moreover, the MSS offers a solution in pure strategies when there is none in Nash terms. In these cases, and similar to the mixed-strategy solution, the MSS comprises a range of prices. In fact, we find that it permits larger price fluctuations than in a mixed-strategy Nash equilibrium. Yet, this price dispersion results from sellers following pure rather than random strategies.[2] The MSS is therefore a unifying concept in that it encompasses all the existing pure- and mixed-strategy Nash equilibria in an intuitive and natural fashion.

By offering a behavioral foundation for oligopoly pricing, this research contributes to the emerging literature on behavioral industrial organization as recently reviewed by Heidheus and Kőszegi (2018). Thus far, research in this field has mainly focused on psychological factors on the demand side. As indicated by Tremblay and Xiao (2020), however, there is increasing attention for analyzing behavioral aspects on the supply side. The application of the MSS stability concept to oligopoly pricing contributes to this research agenda. In particular, it helps shedding light on some real-world economic phenomena such as price dispersion, supply shortages and Edgeworthian price cycles. For instance, we find that the MSS *ceteris paribus* expands with the size of the biggest market player, thereby admitting larger price dispersion. The MSS also provides a rationale for different types of pricing dynamics, including the following two interesting possibilities: (1) A state of *hyper-competition* with corresponding supply shortages, and (2) Edgeworth-like price cycles.

Myopic oligopoly pricing can lead to a state of hyper-competition in which sellers collectively price below the market-clearing price. The logic is roughly as follows. Starting from a market-clearing situation, the biggest market player may have an incentive to hike its price and operate as a monopolist on its contingent demand curve. This creates an incentive for smaller producers to hike their own prices and (approximately) match the price of the largest firm. The biggest supplier can now improve its situation by shaving its price below the prices of its smaller-sized rivals, leaving the latter worse off than in the initial market-clearing situation. This, in turn, makes prices below the market-clearing price a better-response. Myopic oligopoly pricing may therefore induce a state of hyper-competition in which myopic sellers end up setting a price below market-clearing levels. The MSS consequently provides a rationale for rationing, *i.e.*, a situation in which demand exceeds supply.

The MSS moreover offers an explanation for Edgeworth-like price cycles. Edgeworth (1925) pointed out the possibility of producers not being able to meet their demand.[3] If so, prices may

[2] Several authors have argued that mixed strategies might be implausible in the context of oligopoly pricing games. See Friedman (1988) and, more recently, Edwards and Routledge (2019).

[3] Edgeworth (1925) examines price competition under capacity constraints. Edgeworth (1922) considers the equivalent case in which suppliers are not *willing* to meet the demand forthcoming to them. This may occur when the production technology exhibits decreasing returns to scale, for example. Note also that, since the MSS is a static solution concept, it essentially provides an intuitive explanation for particular price patterns following myopic *better-responses*.

never stabilize but instead oscillate indefinitely between some upper and lower bound. More specifically, his analysis hints at the emergence of asymmetric price cycles that essentially consist of two parts. If prices are relatively high, then sellers have an incentive to slightly undercut each other. This leads prices to decrease gradually until a floor is reached (the *price war phase*). At that point, firms have an incentive to hike their price and act as a monopolist on their residual demand curve (the *relenting phase*). This latter incentive comes from (i) the fact that cheaper suppliers will not meet all demand forthcoming to them, and (ii) part of these unserved customers still prefers to buy at the higher price. This in turn provides an incentive for low-priced sellers to hike their price, which induces a new cycle.[4] Several empirical studies have documented the existence of Edgeworth-like price cycles in practice. Eckert (2003) and Noel (2007a,b), for example, provide evidence of such 'sawtooth shape' price patterns in Canadian retail gasoline markets. Among other things, they show that large firms are likely to initiate the relenting phase through a price hike, whereas small firms take the lead in the price war phase. Wang (2008) reports on collusive price cycles in an Australian retail gasoline market.[5] More recently, Zhang and Feng (2011) and Hauschultz and Munk-Nielsen (2020) have shown the presence of Edgeworth-like price patterns in online search-engine advertising and pharmaceutical markets, respectively.

This paper is naturally related to the rich body of theoretical work on capacity-constrained price competition, a literature that basically can be divided in two parts. One focuses on the existence and characterization of the mixed-strategy Nash equilibrium. Such mixed-strategy solutions have been provided by Beckmann (1965), Levitan and Shubik (1972), Osborne and Pitchik (1986), Allen and Hellwig (1986), and Deneckere and Kovenock (1992), amongst others. Another part aims to restore the existence of a pure-strategy Nash equilibrium by rationalizing why residual demand for a high-priced seller would be significantly reduced or even eliminated. For example, Dixon (1990) shows that producers may no longer have an incentive to act as a monopolist on their contingent demand curve when there are cost to turning customers away. Other solutions along this line include Dixon (1992), Tasnádi (1999) and, more recently, Edwards and Routledge (2019). All this work concentrates on Nash solutions and is consequently based on best- rather than better-responses, which is the focus of our analysis.[6]

Capacity-constrained price competition has also been studied in controlled experimental lab-

For an analysis and discussion of Edgeworthian price cycles based on myopic *best-responses*, see De Roos (2012).

[4]Absent capacity constraints, Maskin and Tirole (1988) show how asymmetric price cycles may emerge in equilibrium when firms pick prices sequentially from a grid.

[5]De Roos and Smirnov (2021) analyze theoretically the pricing behavior of a less than all-inclusive price cartel and provide a rationale for collusive Edgeworth-like price cycles. There is also evidence for asymmetric price cycles in European retail gasoline markets. See, *e.g.*, Foros and Steen (2013) and Linder (2018).

[6]It is worth noting that both better- and best-response dynamics are well-known concepts in the game-theoretic literature on learning. A central issue in this work is whether, and under what conditions, better- and best-response adjustments lead to convergence to an equilibrium. See, for example, Milgrom and Roberts (1990), Monderer and Shapley (1996), Friedman and Mezzetti (2001) and Arieli and Young (2016).

oratory settings. Kruse, Rassenti, Reynolds, and Smith (1994), for example, conducted a series of twenty experiments to test the wide variety of theoretical pricing predictions. Among other things, they find a general price decline during the first periods. Towards the middle or the end, however, they observe patterns of upward and downward price swings. This is confirmed by Fonseca and Normann (2013) who also find prices to move up and down for a wide range of capacities. Interestingly, they conclude that:[7]

> ...the data are better explained by Edgeworth-cycle behavior. Not only are average prices closer to the predicted Edgeworth-cycle prices, *but we cannot reject the hypothesis that firms are engaging in some form of myopic price adjustment.*

The new behavioral foundation that we present in this paper indeed provides a rationale for such complex oligopoly pricing dynamics.

The remainder of the paper is organized as follows. The next section presents the model. Section 3 offers a detailed description of the MSS solution concept. Section 4 contains our main findings. These findings are illustrated by means of a linear demand example in Section 5. Section 6 concludes. The proofs are relegated to the Appendix.

2 Model

Consider a homogeneous-good price-setting oligopoly with a finite set of firms: $N = \{1, \ldots, n\}$. Each firm $i \in N$ has a production capacity $k_i > 0$ and produces to order at constant marginal cost, which we normalize to zero.[8] Without loss of generality, we assume that $k_1 \geqslant k_2 \geqslant \ldots \geqslant k_n > 0$ so that firm 1 is the (weakly) largest and firm n is the (weakly) smallest firm in the market. Total industry capacity is given by $K = \sum_{i \in N} k_i$ and $K_{-i} = \sum_{j \in N \setminus \{i\}} k_j$ is the combined production capacity of all firms other than i.

Let market demand be given by the function $D : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$. We make the standard assumptions that $D(.)$ has a finite upper bound $(D(0))$ and is twice continuously differentiable, with $D'(.) < 0$. There is a choke price $\alpha > 0$ and therefore $D(.) = 0$ at prices larger or equal than α. Sellers pick prices simultaneously and we denote supplier i's strategy space by $P_i = [0, \infty)$ so that $P = \prod_{i \in N} P_i$ is the set of all possible strategy profiles.

Since products are homogeneous, consumers prefer to buy from a supplier setting the lowest price. As firms may face capacity constraints, however, it is possible that only part of them will be served in which case higher-priced sellers might still receive demand. To specify individual (residual) demand, let $\Omega(p_i) = \{j \in N | p_j = p_i\}$ and $\Delta(p_i) = \{j \in N | p_j < p_i\}$ denote the set of

[7] Fonseca and Normann (2013, p. 201), italics is ours.

[8] There are some recent contributions that analyze price-quantity competition under the assumption that production precedes sales. See, *e.g.*, Montez and Schutz (2021) and Tasnádi (2020).

firms that price at and below p_i, respectively. Furthermore, let $p_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n)$ indicate the prices of all firms other than i. Demand for firm i's products is then given by $D_i(p_i, p_{-i}) = D(p_i)$ when all its competitors charge a strictly higher price. If there is at least one other seller setting the same price, then its demand is:

$$D_i(p_i, p_{-i}) = \max\left\{ \frac{k_i}{\sum_{j \in \Omega(p_i)} k_j} \left(D(p_i) - \sum_{j \in \Delta(p_i)} k_j \right), 0 \right\}.$$

Finally, if firm i sets the strictly highest price in the industry, its demand is:

$$D_i(p_i, p_{-i}) = \max\left\{ D(p_i) - K_{-i}, 0 \right\}.$$

Thus, (i) customers first visit the lowest-priced seller(s) at the set prices, (ii) at equal prices, demand is allocated in proportion to production capacity, and (iii) rationing is efficient.[9]

Profits are then given by:

$$\pi_i(p_i, p_{-i}) = p_i \cdot \min\{k_i, D_i(p_i, p_{-i})\}, \text{ for all } i \in N.$$

To facilitate the ensuing analysis, we denote firm i's profit by $\pi_i^{\ell}(p_i)$ when p_i is the strictly lowest price and by $\pi_i^h(p_i)$ when p_i is the strictly highest price in the industry. Furthermore, we assume that $\pi_i^h(p_i) = p_i(D(p_i) - K_{-i})$ is strictly concave when there is residual demand for the highest-priced firm (*i.e.*, when $D(p_i) > K_{-i}$) and we write $p_i^* = \arg\max_{p_i} \pi_i^h(p_i)$ to indicate the corresponding **residual profit-maximizing price**.[10] Also, assuming that $K < D(0)$, let $\underline{p}$ be the price for which market demand equals total production capacity ($D(\underline{p}) = K$) and let $\underline{p} = 0$ when $K \geqslant D(0)$. We refer to $\underline{p}$ as the **market-clearing price**.[11] Figure 1 provides a graphical illustration.

The next result relates an individual price choice to the market-clearing price $\underline{p}$. Specifically, it shows that if industry capacity is sufficiently small (*i.e.*, $K < D(0)$), then $\pi_i(p_i, p_{-i}) = p_i k_i$ provided that $D(p_i) \geqslant K$. That is, a firm produces at capacity whenever there is excess demand at the set price.

Lemma 1. *If* $0 < p_i \leqslant \underline{p}$*, then* $\pi_i(p_i, p_{-i}) = p_i k_i$*, for all* $i \in N$.

As is well-known, existence of a pure-strategy Nash equilibrium in capacity-constrained pricing games critically depends on available production capacities. If capacities are large enough in

[9]Such a surplus maximizing scheme is also used by Levitan and Shubik (1972), Kreps and Scheinkman (1983), Osborne and Pitchik (1986) and Edwards and Routledge (2019), amongst others.

[10]We assume strict concavity for analytical convenience. Strictly speaking, it would be sufficient to impose a weaker requirement such as single-peakedness.

[11]To economize on notation, we refer to $\underline{p}$ sometimes as a price and sometimes as a price profile with all firms pricing at $\underline{p}$. It is clear from the context what is meant. Note further that, since production is to order, there are in fact many market-clearing prices in this model.

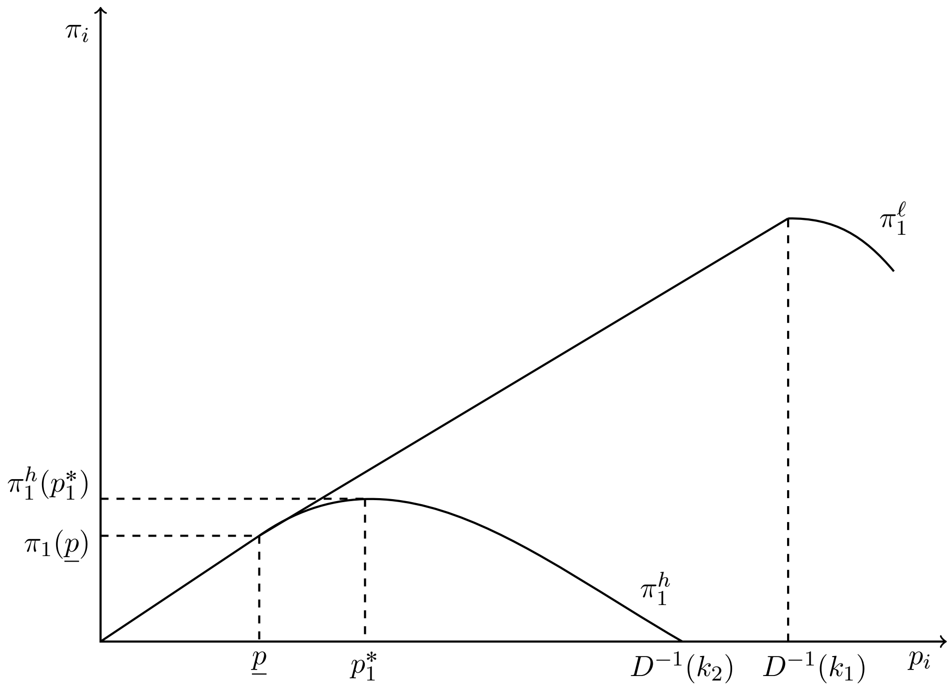

Figure 1: An illustration of firm 1's profit function when $n = 2$.

relation to market demand, then there is a symmetric 'Bertrand-type' pure-strategy solution in which all sellers price at cost. A pure-strategy equilibrium also exists when capacities are sufficiently small in the sense that market demand is elastic at all prices above $\underline{p}$.[12] In that situation, all suppliers charge the same market-clearing price and produce at capacity. Finally, for an intermediate range of production capacities, there is no pure-strategy Nash solution. There does exist a mixed-strategy equilibrium, however, which will be elaborated on in Section 4.3 below.

3 Solution Concept

In the following, we do not take the standard Nash approach. Instead, we employ the concept of *Myopic Stable Set* which is based on the idea that sellers may simply aim to improve upon their situation and not necessarily maximize their profits. In this section, we introduce this equilibrium concept in detail.

Consider a price profile $p = (p_1, \ldots, p_n) \in P$. We say that an alternative price profile $p' \in P$ dominates p when there is a firm that can unilaterally deviate to p' and earn a higher profit under p' than under p. That is, given the strategy profile p, this firm has a better-reply since it can myopically improve itself by inducing price profile p'.

Definition 1. *Let $p, p' \in P$ be two price profiles. The price profile p' dominates p, $p' \succ p$, if there exists a firm $i \in N$ such that $\pi_i(p') > \pi_i(p)$ and $p'_{-i} = p_{-i}$.*

[12] For a detailed analysis of this possibility, see Tasnádi (1999).

Next, given some price profile $p \in P$, we write $f(p)$ to describe the subset of P consisting of all dominating price profiles in conjunction with p:

$$f(p) = \{p\} \cup \{p' \in P | p' \succ p\}.$$

Given f, let the set of pure-strategy Nash equilibria be denoted by:

$$\text{NE} = \{p \in P | f(p) = p\}.$$

To capture the better-reply dynamics that can be generated by the firms, we define the κ-fold iteration $f^{\kappa}(p)$ as the subset of P that contains all the price profiles obtained by a composition of dominance correspondences of length $\kappa \in \mathbb{N}$. Thus, $p' \in f^{\kappa}(p)$ when there is a $p'' \in P$ such that $p' \in f(p'')$ and $p'' \in f^{\kappa-1}(p)$. Note further that if $\kappa \leqslant t$, then $f^{\kappa}(p) \subseteq f^{t}(p)$, for all $\kappa, t \in \mathbb{N}$. We indicate the set of prices that can be reached from p by a finite number of dominations by $f^{\mathbb{N}}(p)$:

$$f^{\mathbb{N}}(p) = \bigcup_{\kappa \in \mathbb{N}} f^{\kappa}(p).$$

Given $p', p \in P$, we say that a price profile p' asymptotically dominates p when, starting from p, it is possible to get arbitrarily close to p' through a finite number of myopic improvements.

Definition 2. *A price profile $p' \in P$ asymptotically dominates $p \in P$ if there exists a number $\kappa \in \mathbb{N}$ and a price profile $p'' \in f^{\kappa}(p)$ such that $||p' - p''|| < \epsilon$, for all $\epsilon > 0$.*

We denote by $f^{\infty}(p)$ the set of all strategy profiles in P that asymptotically dominate p. Formally,

$$f^{\infty}(p) = \{p' \in P | \forall \epsilon > 0,\ \exists \kappa \in \mathbb{N}, \exists p'' \in f^{\kappa}(p) : ||p' - p''|| < \epsilon\}.$$

Notice that the set $f^{\infty}(p)$ coincides with the closure of the set $f^{\mathbb{N}}(p)$.

We now have all the ingredients available to define the Myopic Stable Set (MSS) for the capacity-constrained pricing game:

Definition 3. *Let $G = \{N, (P_i, \pi_i)_{i \in N}\}$ be a capacity-constrained pricing game as specified in Section 2. The set $M \subseteq P$ is a Myopic Stable Set when it is closed and satisfies the following three conditions:*

i. ***Deterrence of External Deviations:*** *For all $p \in M$, $f(p) \subseteq M$.*

ii. ***Asymptotic External Stability:*** *For all $p \notin M$, $f^{\infty}(p) \cap M \neq \emptyset$.*

iii. ***Minimality:*** *There is no closed set $M' \subsetneq M$ that satisfies conditions i and ii.*

Suppose there is a set M of myopically stable price profiles. 'Deterrence of External Deviations' means that no firm can profitably deviate to a price profile outside M. 'Asymptotic External Stability' requires that any price profile outside M is asymptotically dominated by a price profile in M. Hence, from any price profile outside M it is possible to get arbitrarily close to one in M by a finite number of myopic improvements. Finally, 'Minimality' means that there is no smaller (closed) set for which the first two conditions are met. Roughly speaking, the MSS can thus be pictured as a set of price profiles that, once entered through the dominance dynamics, is never left.

4 Results

For normal form games, Demuynck, Herings, Saulle and Seel (2019a) prove the existence of a unique MSS when the strategy space is compact and the payoff functions are continuous. The continuity assumptions are not satisfied in the capacity-constrained pricing model, however. In this section, we show that this game also possesses a unique MSS for any given level of capacities. Moreover, we characterize this solution and compare it to the set of pure-strategy Nash equilibria as well as to the support of the mixed-strategy Nash equilibrium. Among other things, we establish that the MSS encompasses all existing Nash equilibrium solutions.

4.1 Pricing Equilibria with Large or Small Capacities

We begin with exploring the relationship between the MSS and the set of pure-strategy Nash equilibria. Towards that end, denote a subset of sellers $S \subseteq N$ *minimal* when $\sum_{j \in S \setminus \{i\}} k_j \geqslant D(0)$, for all $i \in S$. That is, each combination S has sufficient capacity to meet market demand at a zero price when a member leaves the coalition. Let us now present conditions under which the set of pure-strategy Nash equilibria NE is nonempty.[13]

- If $K_{-1} \geqslant D(0)$, then
  $\text{NE} = \left\{p \in P | p \in \prod_{i \in S} \{0\} \times \prod_{i \in N \setminus S} [0, \infty)\right\}.$
- If $K \leqslant D(p_1^*)$, then
  $\text{NE} = \{p \in P | p_i = \underline{p} > 0, \text{ for all } i \in N\}.$

Proposition 1. *Let G be a capacity-constrained pricing game as specified in Section 2. The set NE is the set of pure-strategy Nash equilibria of G.*

Simply put, there are two types of pure-strategy Nash equilibria. If industry capacity is sufficiently large, then there is a set of pure-strategy solutions, all of which have firms making zero economic profit. One solution in this case is the symmetric 'Bertrand-type' pure-strategy equilibrium in which all firms price at cost. There are also many asymmetric equilibria in which part

[13] Vives (1986) provides conditions for non-emptiness of the set of symmetric pure-strategy Nash equilibria under a surplus maximizing scheme. In Proposition 1, we additionally admit asymmetric pure-strategy Nash equilibria.

of the firms price above cost and have no demand. If aggregate capacity is sufficiently small, then there is a symmetric pure-strategy Nash equilibrium in which each firm sets its price equal to the market-clearing price.

To better illustrate the relation between the MSS and the set of pure-strategy Nash equilibria, we now introduce the so-called *weak improvement property*.

Definition 4. *A normal form game satisfies the weak improvement property when* $NE \neq \emptyset$ *and* $f^{\infty}(p) \cap NE \neq \emptyset$ *for each price profile* $p \notin NE$.

A normal form game possesses the *weak improvement property* when any non-Nash equilibrium strategy profile converges to a Nash equilibrium through a finite sequence of myopic improvements. Demuynck, Herings, Saulle and Seel (2019a) extend previous results by Monderer and Shapley (1996), Friedman and Mezzetti (2001) and Dindõs and Mezzetti (2006) by showing that supermodular games (Friedman and Mezzetti, 2001) and pseudo-potential games (Dubey, Haimanko and Zapechelnyuk , 2006), including games of strategic complements or substitutes with aggregation (*e.g.*, Cournot oligopolies), exhibit the weak improvement property. The capacity-constrained pricing model does not belong to any of the aforementioned game classes, however. Nevertheless, we establish with the next proposition that this type of games also exhibit the weak improvement property.

Proposition 2. *Let G be a capacity-constrained pricing game as specified in Section 2. If the set of pure-strategy Nash equilibria NE is nonempty, then this game exhibits the weak improvement property.*

This result states that any price profile that is not a pure-strategy Nash equilibrium is asymptotically dominated by the pure-strategy solution(s). That is, from any price profile not in NE it is possible to get arbitrarily close to a pure-strategy equilibrium by a finite number of myopic improvements.

Using the preceding results, we now show that the set of pure-strategy equilibria coincides with the MSS whenever the former is nonempty.

Theorem 1. *Let G be a capacity-constrained pricing game as specified in Section 2. If the set of pure-strategy Nash equilibria NE is nonempty, then NE is the unique Myopic Stable Set.*

The above approach captures the potential pricing dynamics of the game. This includes prices that may emerge under myopic improvements and in particular also the pure-strategy Nash equilibrium prices. More specifically, following Proposition 2, there is a price path of myopic improvements from any non-Nash price profile to a pure-strategy Nash profile. Once the pricing dynamics enters the set NE, however, there is no way out. Indeed, by the very nature of a Nash equilibrium, none of the sellers can profitably deviate to a price profile outside NE. Combining these two forces yields the result of Theorem 1; that is, there is a unique MSS that coincides with the set of pure-strategy Nash equilibria NE.

4.2 Myopic Stability with Intermediate Capacities

As indicated above, the set of pure-strategy Nash equilibria is empty in the capacity-constrained pricing model when capacities are in an intermediate range (*i.e.*, when $D(p_1^*) < K < D(0) + k_1$). We now proceed with analyzing the MSS for these types of cases. To that end, we first introduce two notions that are useful in the ensuing analysis; the **iso-profit price** and the **hyper-competitive price**.

Definition 5. *For each firm $i \in N$, the* ***iso-profit price*** *is:*

$$\overline{p}_i = \begin{cases} \min\left\{p_i \in P_i | \pi_i^h(p_i) = \underline{p} \cdot k_i \textit{ with } p_i \neq \underline{p}\right\} & \textit{if } D(0) > K_{-i} > D(p_i^*) - k_i, \\ \underline{p} & \textit{otherwise.} \end{cases}$$

Given that all its competitors charge a lower price, the iso-profit price of firm i is the lowest price above the market-clearing price $\underline{p}$ for which it receives the same profit as when it would price at $\underline{p}$. It should be emphasized that the iso-profit price differs from the market-clearing price only when the following two conditions hold. First, firm i must face residual demand for some prices (*i.e.*, $D(0) > K_{-i}$). Second, its residual profit-maximizing price must exceed the market-clearing price, which requires a sufficiently large capacity (*i.e.*, $k_i > D(p_i^*) - K_{-i}$). If either of the two conditions is violated, then the iso-profit price coincides with the market-clearing price. A detailed analysis of the various scenarios is provided in Lemma 2 below.

Definition 6. *For each firm $i \in N$, the* ***hyper-competitive price*** *is:*

$$\widetilde{p}_i = \left\{\min\{p_i \in P_i | \pi_i^h(\overline{p}_1) = p_i \cdot k_i\right\}.$$

In words, the hyper-competitive price is the lowest price for which a firm obtains the same profit as when it sets the iso-profit price of the largest seller, given that this iso-profit price is the strictly highest price in the market. A graphical illustration of these two concepts is provided in Figure 2.

Let us now present several results that establish some useful properties with regards to the iso-profit and hyper-competitive prices. Part (*i*) of Lemma 2 gives conditions under which the iso-profit price exceeds the market-clearing price and shows that the iso-profit price is increasing in capacity. Part (*ii*) and (*iii*) describe when the iso-profit price coincides with the market-clearing price, which is the case when the firm is sufficiently small. Part (*ii*) captures the possibility that a firm faces residual demand, but where its residual profit-maximizing price is lower than the market-clearing price. Part (*iii*) shows the possibility that a firm faces no residual demand at any price. In all cases, it holds that the iso-profit price of the largest firm is strictly positive and above the market-clearing price.

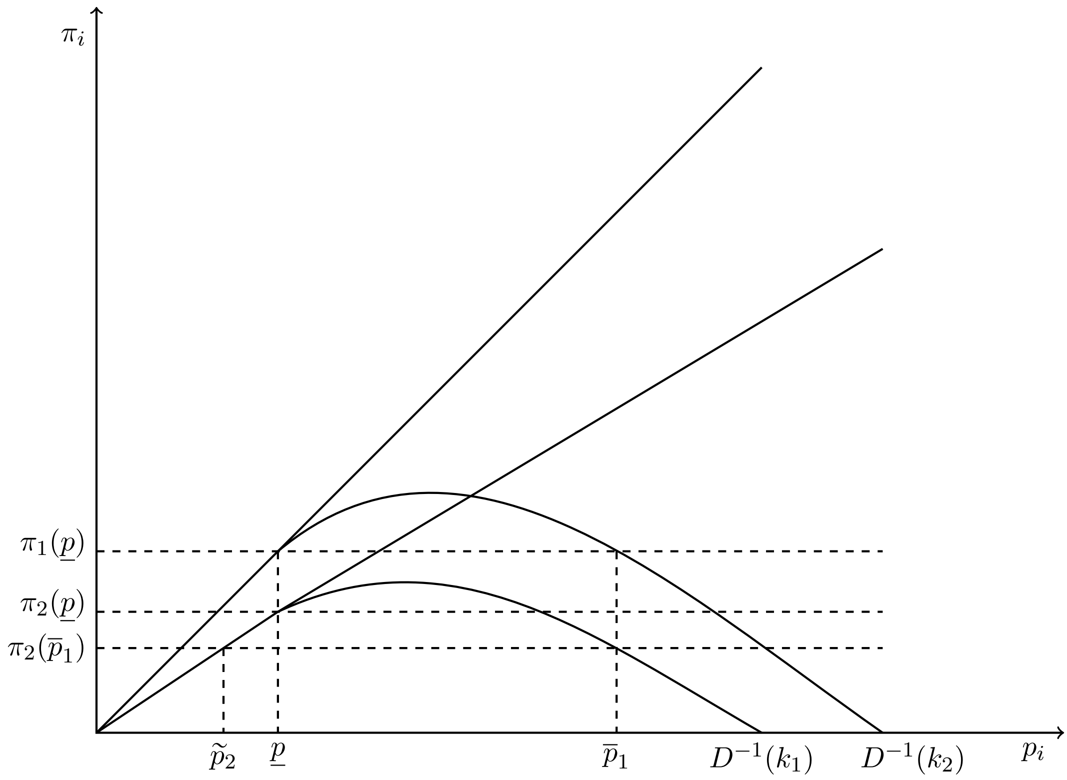

Figure 2: An illustration of the iso-profit price $\overline{p}_1$ and the hyper-competitive price $\widetilde{p}_2$.

Lemma 2. *Suppose there is no pure-strategy Nash equilibrium.*
(i) If $D(0) > K_{-i} > D(p_i^*) - k_i$, for all $i \in N$, and $k_i > k_j$, for any $i, j \in N$, then $\overline{p}_i > \overline{p}_j > \underline{p} \geqslant 0$.
(ii) For all $i \in N \backslash \{1\}$, if $k_i \leqslant D(p_i^*) - K_{-i}$, then $\overline{p}_i = \underline{p}$. For firm 1, $\overline{p}_1 > \underline{p}$.
(iii) If $D(0) \leqslant K_{-m}$ with $n \geqslant m > 1$, then $\overline{p}_i = \underline{p} = 0$ for each firm $i = m, m+1, m+2, ..., n$ weakly smaller than m.

The next lemma focuses on the hyper-competitive price. Part (i) of Lemma 3 shows that hyper-competitive prices are (weakly) below the market-clearing price whenever the latter is strictly positive. Part (ii) establishes a positive relationship between the hyper-competitive price and firm size. Part (iii) states that the hyper-competitive price is zero for a highest-priced firm not facing residual demand.

Lemma 3. *Suppose there is no pure-strategy Nash equilibrium.*
(i) If $K < D(0)$ and each firm $i \in N \backslash \{1\}$ is strictly smaller than firm 1, then $0 < \widetilde{p}_i < \underline{p}$ and $\widetilde{p}_1 = \underline{p}$.
(ii) If $K < D(0)$ and $k_i > k_j$, then $\widetilde{p}_i > \widetilde{p}_j$, for all $i, j \in N$.
(iii) If $D(0) \leqslant K_{-i}$, then $\widetilde{p}_i = \underline{p} = 0$, for all $i \in N$.

Finally, Lemma 4 relates the iso-profit and hyper-competitive prices to a firm's profit. Part (i) of Lemma 4 states that all sellers other than the largest one(s) can profitably raise their price from the market-clearing price $\underline{p}$ to some higher price weakly below the iso-profit price of firm

1. Part (*ii*) complements this by showing that if firm 1 prices below $\overline{p}_1$ and any highest priced seller i other than firm 1 prices at $\overline{p}_1$, then the latter can myopically improve by reducing its price below the market-clearing price (see Figure 2).

Lemma 4. *Suppose there is no pure-strategy Nash equilibrium. For all $i \in N \backslash \{1\}$:*
(i) If $p_1 = \overline{p}_1$, then $\pi_i(p_i, p_{-i}) > \pi_i(\underline{p})$ for $p_i \in (\underline{p}, \overline{p}_1)$.
(ii) If $p_1 < \overline{p}_1$, then $\pi_i(p_i, p_{-i}) > \pi_i^h(\overline{p}_1)$ for $p_i \in (\tilde{p}_i, \underline{p}]$.

Now that we have introduced the iso-profit and hyper-competitive prices as well as some of the corresponding properties, we can analyze the MSS when capacities are in an intermediate range. Specifically, we show in the following that the MSS is given by:

$$M = \left\{ p \in P \;\middle|\; \tilde{p}_i \leqslant p_i \leqslant \overline{p}_1, \;\; \forall i \in N \right\}. \tag{1}$$

Theorem 2. *Let G be a capacity-constrained pricing game as specified in Section 2. If the set of pure-strategy Nash equilibria NE is empty, then M as given in (1) is the unique Myopic Stable Set.*

To provide some intuition for this set solution, consider Figure 3 which gives a graphical illustration. In Figure 3, the MSS is given by the shaded area. In principle, this area admits different types of pricing dynamics. The red arrows represent one particular better-response price path. Starting at point 'c' firms are undercutting each other's prices until point 'a'. At 'a', firm 1 hikes its price to $\overline{p}_1$ at point 'b'. This, in turn, makes it a better-response for firm 2 to price slightly below $\overline{p}_1$. As this example illustrates, the MSS naturally captures Edgeworth-like price cycles and consequently provides a clear rationale for such 'sawtooth shape' price patterns.

Another striking possibility is that the smaller supplier may find it in his interest to set a price below the market-clearing price. Such a scenario is depicted in Figure 4. As before, suppose that both sellers set a price close to the market-clearing price at 'a'. By hiking its price, firm 1 may then induce a price profile at 'b'. This, in turn, may trigger firm 2 to slightly undercut firm 1's price, which leads to a price profile around 'c'. In particular, firm 2 may set a price $p_2' \in (\overline{p}_2, \overline{p}_1]$. Slightly undercutting p_2' may then constitute a better-response for firm 1, which results in a price profile at, say, 'd'. Yet, in that case it is profitable for firm 2 to reduce its price to $p_2'' \in (\tilde{p}_2, \underline{p}]$, which is below the market-clearing price. This possibility is also illustrated in Figure 2 above.

The MSS therefore provides a rationale for 'price wars' where all but the largest seller set a price below the market-clearing price.

Corollary 1. *If each firm $i \in N \backslash \{1\}$ is strictly smaller than firm 1, then there exists a price profile $p \in M$ with $p_1 = \underline{p}$ and $p_i \in [\tilde{p}_i, \underline{p})$.*

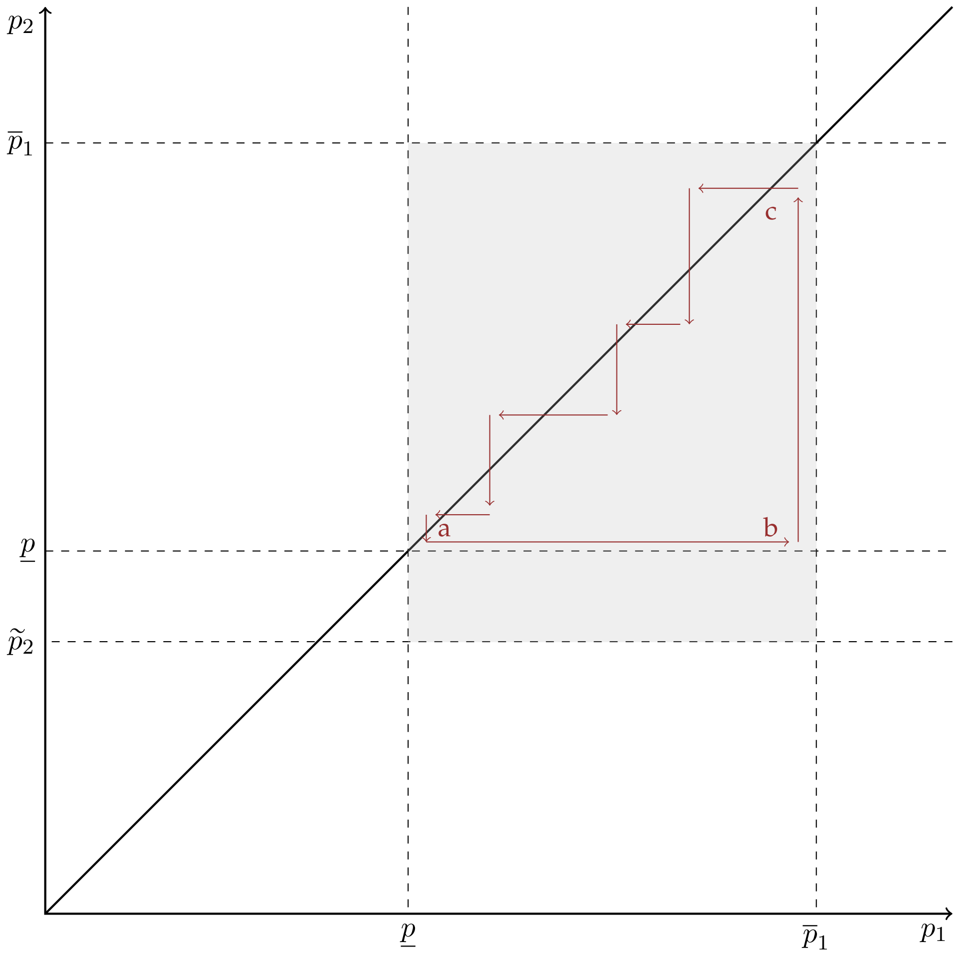

Figure 3: An illustration of the Myopic Stable Set when $n = 2$ and $k_1 > k_2$.

Note that such hyper-competitive price profiles are Pareto-dominated in that all producers would be better-off (and no one worse off) when pricing weakly above the market-clearing price. Moreover, this result highlights the possibility of an equilibrium shortage, *i.e.*, a situation in which market demand exceeds aggregate supply.

Finally, the next result relates the MSS M to the size distribution of firms.

Proposition 3. *Assume $K < D(0)$. An increase in k_1, ceteris paribus, leads to larger price dispersion as reflected by a bigger Myopic Stable Set M.*

Thus, in terms of comparative statics, expansion by the largest seller enables larger price dispersion. The reason is that an increase in firm 1's capacity leads to a higher iso-profit price $\overline{p}_1$, which implies a reduction of profits for firms pricing at $\overline{p}_1$. This, in turn, creates a downward pressure on the hyper-competitive price since, by definition, this is the lowest price for which a seller obtains the same profit as when it sets the iso-profit price of the biggest market player.

To sum up, Theorems 1 and 2 establish the existence and characterization of a unique Myopic

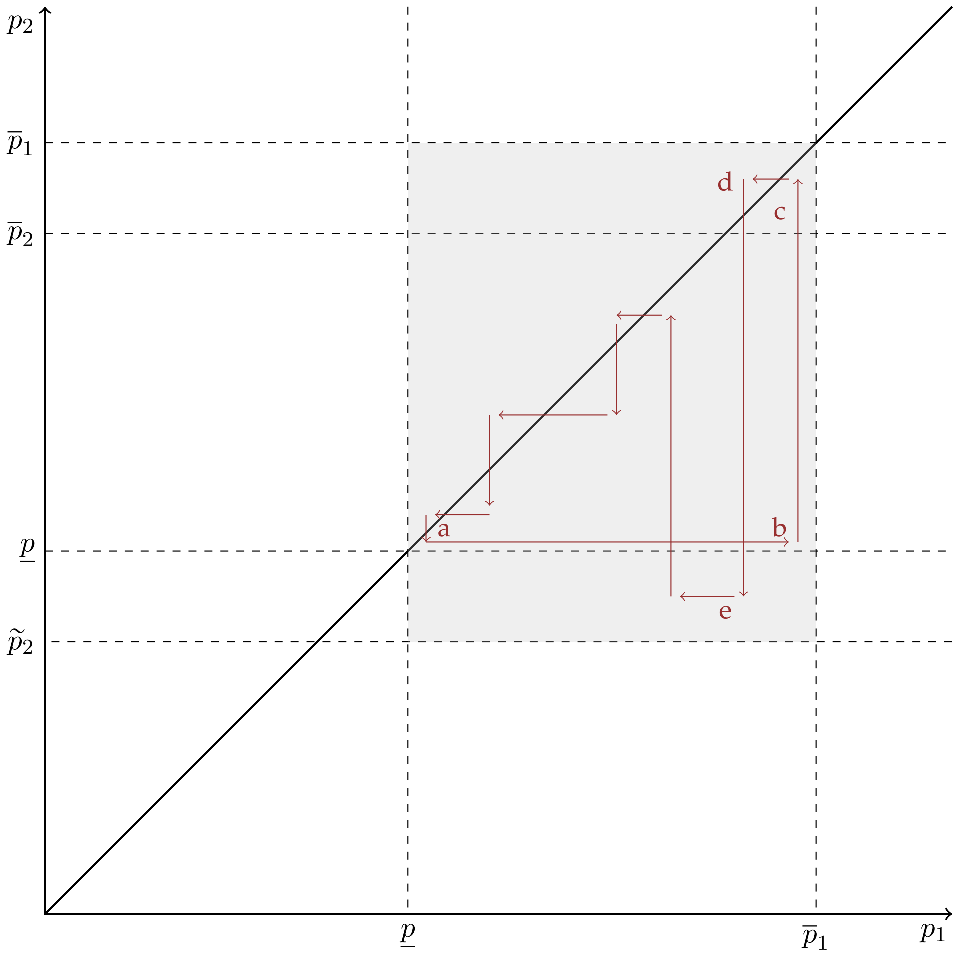

Figure 4: An illustration of the Myopic Stable Set when $n = 2$ and $k_1 > k_2$.

Stable Set for any given level of capacities. In particular, they offer a pure-strategy solution when there is none in Nash terms. Let us now conclude this section by relating the MSS as derived above to the mixed-strategy Nash equilibrium.

4.3 Equilibria in Pure and Mixed Strategies

The existing literature has repeatedly shown that there is a Nash equilibrium in mixed strategies under fairly weak assumptions.[14] Specifically, since in the above capacity-constrained pricing model demand is continuously decreasing and residual profit functions are continuous and strictly concave, there exists an equilibrium in mixed strategies without 'holes' when capacities are within an intermediate range, *i.e.*, when $D(p_1^*) < K < D(0) + k_1$.[15] Let us now consider this mixed-strategy equilibrium in more detail.

To begin, recall that there exists a unique residual profit-maximizing price p_i^* for each firm

[14] Maskin (1986) provides a general analysis and discussion of these existence conditions.
[15] See, *e.g.*, Deneckere and Kovenock (1992) and, more recently, Tasnádi (2020).

$i \in N$. By construction of the contingent demand functions $D(p_i) - K_{-i}$, it then follows that $p^m \geqslant p_1^* \geqslant p_2^* \geqslant \ldots \geqslant p_n^*$, where p^m is the monopoly price. Note that p^m approaches p_1^* when K_{-1} approaches zero.[16] Since none of the sellers has an interest in charging a price in excess of p_1^*, this price constitutes the upper bound of the mixed-strategy support. Let the lower bound be indicated by $\widehat{p}_i$, where $\widehat{p}_i \neq p_i^*$ is the price solving

$$p_i^* \left[D(p_i^*) - K_{-i}\right] = \min\left\{\widehat{p}_i k_i, \widehat{p}_i \cdot D\left(\widehat{p}_i\right)\right\}.$$

The mixed-strategy support is therefore given by:

$$\mathcal{K} = \prod_{i \in N} \left[\widehat{p}_i, p_1^*\right] \subset P.$$

Let us now relate $\mathcal{K}$ to the MSS. Recall that in this case there is a unique MSS given by the set M (Theorem 2). The next result shows that the MSS permits larger price fluctuations in comparison to the mixed-strategy equilibrium.

Theorem 3. *Let G be a capacity-constrained pricing game as specified in Section 2. If the set of pure-strategy Nash equilibria NE is empty, then $\mathcal{K} \subset M$.*

The intuition underlying this finding is as follows. Regarding the upper bound, with random strategies no firm puts mass on prices above the maximizer of its (residual) profit function. By contrast, the MSS permits such prices since prices in excess of the maximizer may still constitute a better-response. The story is to some extent similar for the lower bounds. To see this, notice that in a mixed-strategy equilibrium none of the sellers prices below $\underline{p}$ because either it can sell its entire capacity at $\underline{p}$ or $\underline{p} = 0$. As there are no pure-strategy Nash equilibria in this case, there is at least one firm that would be willing to hike its price when all firms price at $\underline{p}$. It can be easily verified that this holds for the largest firm. Since the higher-priced firm has residual demand, the lower-priced firms are capacity-constrained. This provides an incentive to also raise their prices, which in turn implies that no seller puts mass on prices weakly below $\underline{p}$. By contrast, following the definition of the MSS upper bound $\overline{p}_1$, $\underline{p}$ must be part of the MSS since profits are the same at both prices. In fact, and as illustrated in Figure 2 and Figure 3 above, it is quite possible that one or more sellers have a better-response below $\underline{p}$.

In sum, the above analysis shows that myopic sellers set the same price as their profit-maximizing counterparts when production capacities are either 'large' or 'small'. For an intermediate range

[16] Since for any $i, j \in N$, p_i^* and p_j^* are such that

$$D(p_i^*) + p_i^* D'(p_i^*) = K_{-i}$$

and

$$D(p_j^*) + p_j^* D'(p_j^*) = K_{-j}$$

$K_{-i} < K_{-j} \Leftrightarrow p_i^* > p_j^*$ by concavity of firms' (residual) profit functions.

of capacities, the set of mixed-strategy profiles is a subset of the MSS. The MSS thus encompasses all Nash solutions. The next section provides an example illustrating these findings.

5 Example

Let us examine a Bertrand-Edgeworth duopoly with linear market demand: $D(p) = 1 - p$. Demand for the products of firm i, $i = 1, 2$ and $i \neq j$, is then described by the following demand structure:

$$D_i(p_i, p_j) = \begin{cases} 1 - p_i & \text{if } p_i < p_j, \\ \frac{k_i}{k_i + k_j}(1 - p_i) & \text{if } p_i = p_j, \\ \max\{0, 1 - p_i - k_j\} & \text{if } p_i > p_j. \end{cases} \tag{2}$$

It is assumed that $k_1 > k_2$ so that firm 1 is strictly larger in terms of capacity. Below, we derive the MSS for the entire range of production capacities and compare it to the standard Nash solution.

Let us begin with the situation where capacities are 'large' so that a pure-strategy Nash equilibrium exists. Specifically, this is the case when each seller can serve the whole market at the competitive price, *i.e.*, when $k_1 > k_2 \geqslant 1$. The Nash equilibrium is then such that both firms charge a price equal to marginal cost and therefore (by Theorem 1):

$$M = NE = \{(0, 0)\}.$$

A pure-strategy Nash equilibrium also exists when capacities are sufficiently small. Specifically, this is true when $k_1 \leqslant \underline{k}_1$, where $\underline{k}_1$ solves the following equality:

$$\underline{k}_1 = D(p_1^*) - k_2. \tag{3}$$

In our linear example, firm 1's residual profit-maximizing price is:

$$p_1^* = \frac{1}{2}(1 - k_2).$$

Substituting in (3) and rearranging gives the threshold value $\underline{k}_1 = (1 - k_2)/2$. Thus, for $k_1 \leqslant \underline{k}_1$ (or, equivalently, $k_2 \leqslant 1 - 2k_1$), there is a pure-strategy solution for which the market clears. Moreover, by Theorem 1, this pure-strategy Nash equilibrium coincides with the MSS:

$$M = NE = \{(1 - k_1 - k_2, 1 - k_1 - k_2)\}.$$

For the capacity ranges specified above there exists no nondegenerate mixed-strategy Nash equilibrium. Let us now turn to the possibility where there is a nondegenerate Nash equilibrium in mixed strategies. This is the case when $k_1 > \underline{k}_1$ and $1 > k_2 > 1 - 2k_1$. To determine the lower bound of the mixed-strategy support, notice that firm 1 is indifferent between being the high- and the low-priced firm when:[17]

$$\pi_1^h\left(p_1^*\right) = p_1^* \cdot (1 - p_1^* - k_2) = \frac{1}{4}\left(1 - k_2\right)^2 = \pi_1^\ell\left(\widehat{p}_1\right) = \widehat{p}_1 \cdot \min\{k_1, 1 - \widehat{p}_1\},$$

so that

$$\widehat{p}_1 = \frac{1}{4k_1}\left(1 - k_2\right)^2 \text{ when } k_2 \leqslant 1 - \sqrt{1 - (2k_1 - 1)^2},$$

and

$$\widehat{p}_1 = \frac{1}{2} - \frac{1}{2}\sqrt{2k_2 - k_2^2} \text{ when } 1 - \sqrt{1 - (2k_1 - 1)^2} < k_2 \leqslant 1.$$

The mixed-strategy support of this Bertrand-Edgeworth game is therefore given by:

$$[\widehat{p}_1, p_1^*] = \left[\frac{\left(1 - k_2\right)^2}{4k_1}, \frac{1 - k_2}{2}\right] \text{ or } [\widehat{p}_1, p_1^*] = \left[\frac{1}{2} - \frac{1}{2}\sqrt{2k_2 - k_2^2}, \frac{1 - k_2}{2}\right],$$

depending on the capacity levels.

Let us now derive the MSS for this intermediate range of capacities. By Theorem 2, we know that the *upper bound* of the MSS is the price $\overline{p}_1$ solving

$$\pi_1^h\left(\overline{p}_1\right) = \overline{p}_1\left[D(\overline{p}_1) - k_2\right] = \pi_1\left(\underline{p}\right),$$

whereas the *lower bound* for the largest firm (firm 1) is given by:

$$\underline{p} = \max\left\{0, D^{-1}(k_1 + k_2)\right\}. \tag{4}$$

We therefore need to distinguish two cases.

The first is where combined production capacity is sufficiently large to serve the whole market at the competitive price: $k_1 + k_2 \geqslant 1$. In this case, $\underline{p} = 0$ and $\overline{p}_1$ is the price that solves

$$\pi_1^h\left(\overline{p}_1\right) = \overline{p}_1\left[D(\overline{p}_1) - k_2\right] = \pi_1\left(\underline{p}\right) = 0,$$

which implies

$$D(\overline{p}_1) = k_2, \text{ and therefore } \overline{p}_1 = D^{-1}(k_2) = 1 - k_2.$$

In this case, $\underline{p} = 0$ is also the MSS lower bound for firm 2. Thus, when $1 > k_1 \geqslant 1 - k_2$, the MSS is symmetric and given by:

$$M = \left[0, D^{-1}(k_2)\right] \times \left[0, D^{-1}(k_2)\right] = [0, 1 - k_2] \times [0, 1 - k_2].$$

[17] Note that in this duopoly example $\widehat{p}_1$ exceeds $\widehat{p}_2$, so that $\widehat{p}_1$ is the lower bound of the mixed-strategy support. For a detailed analysis, see Deneckere and Kovenock (1992).

Observe that an increase in k_2 lowers the upper bound of the MSS since it reduces the residual demand for firm 1's products when it is the high-priced seller. This, in turn, makes that the 'optimal high price' is lower leaving fewer prices that qualify as a better-response.

Now consider the other possibility where $k_1 + k_2 < 1$ so that $\underline{p} = 1 - k_1 - k_2 > 0$. In this case, the upper bound of the MSS is obtained by solving

$$\underline{p} \cdot (1 - \underline{p} - k_2) = \overline{p}_1 \cdot (1 - \overline{p}_1 - k_2),$$

which is equivalent to

$$k_1 \cdot (1 - k_1 - k_2) = \overline{p}_1 \cdot (1 - \overline{p}_1 - k_2),$$

and therefore $\overline{p}_1 = k_1$. In contrast, the lower bound of the MSS differs across firms. For firm 1 this lower bound is the same as in the previous case. For firm 2, however, it is the price $\widetilde{p}_2$ given by:

$$\pi_2^h(\overline{p}_1) = \overline{p}_1 [D(\overline{p}_1) - k_1] = \widetilde{p}_2 \cdot k_2$$

with $\widetilde{p}_2 \neq \overline{p}_1$. This price solves $k_1(1 - 2k_1) = \widetilde{p}_2 k_2$, which gives $\widetilde{p}_2 = k_1(1 - 2k_1)/k_2 < \underline{p}$. Thus, when $1 - k_2 > k_1 > (1 - k_2)/2$, the MSS is asymmetric and given by:

$$M = [\underline{p}, \overline{p}_1] \times [\widetilde{p}_2, \overline{p}_1] = [1 - k_1 - k_2, k_1] \times [k_1(1 - 2k_1)/k_2, k_1].$$

Notice that k_1 has a positive impact on the size of the MSS since it reduces the lower bounds while increasing the upper bounds (Proposition 3).[18] Also, and in contrast to the previous case, k_2 has a positive impact on the size of the MSS by reducing the lower bounds.

To conclude, let us now compare the range of the mixed-strategy support $[\widehat{p}_1, p_1^*]$ with the price range of the MSS for all (relevant) capacity levels. Figures 5 and 6 provide a graphical illustration.

In Figure 5, the MSS is depicted by the solid (thick) black line for every level of k_1 and k_2 (expressed as a function of k_1, which in this figure is $k_2(k_1) = \frac{9}{10} \cdot k_1$). Starting from the left, for sufficiently small capacities there is a pure-strategy Nash equilibrium that coincides with the MSS (Theorem 1). For this specific example, this is true as long as $k_1 \leqslant \underline{k}_1 \approx 0.345$.[19] At that point, the market-clearing price $\underline{p}$ (indicated by the thin solid line) starts to fall below the maximizer of π_1^h (indicated by the straight dashed line). This provides an incentive for firms to hike their price and become the high-priced firm.

The increase in capacities not only undermines the existence of a pure-strategy Nash solution; it also widens the range of better-responses. To see this, recall that $\overline{p}_1$ is the lowest price above $\underline{p}$

[18]Note that $\tilde{p}_2$ is decreasing in k_1 for $k_1 > 1/4$, which holds true in this case.

[19]This threshold value can be computed by using $k_1 = \underline{k}_1 = \frac{(1-k_2)}{2}$ and $k_2 = \frac{9}{10}k_1$. Combining gives $k_1 \approx 0.345$ and $k_2 \approx 0.31$.

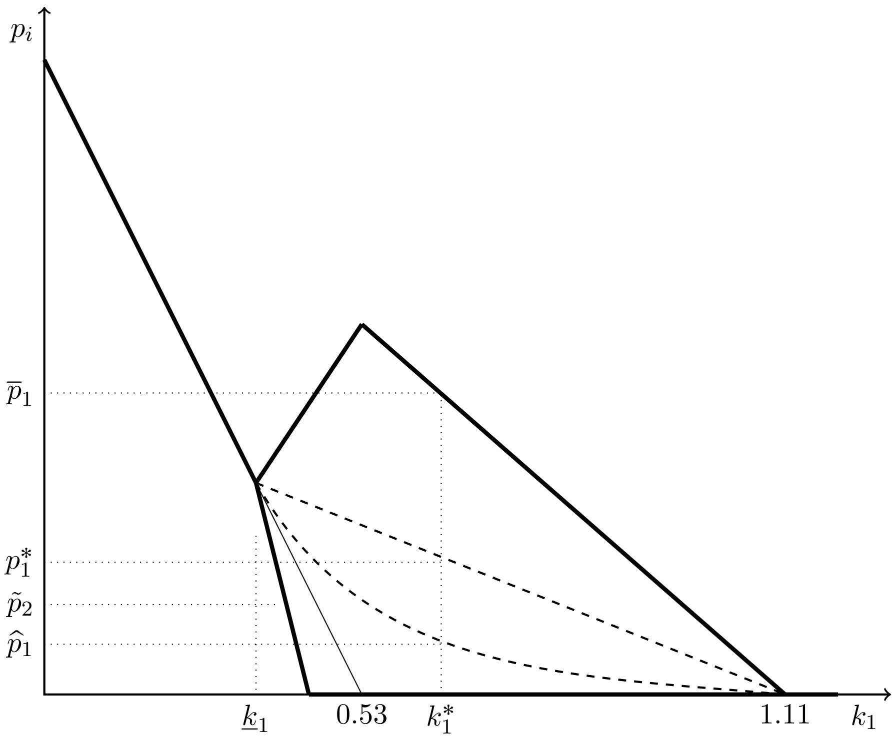

Figure 5: Myopic Stable Set in a linear demand duopoly with $k_2 = \frac{9}{10}k_1$.

for which firm 1 obtains the same profit. Therefore, and due to the fact that the profit function is strictly concave and has a unique maximum (see Figure 1 and Figure 2), the MSS upper bound ($\overline{p}_1$) is increasing and the MSS lower bound ($\tilde{p}_2$) is decreasing in the gap between p_1^* and $\underline{p}$. This range of better-response prices is rising until $k_1 \approx 0.53$, the capacity level at which $\underline{p}$ becomes zero.[20] Beyond that point, $\underline{p}$ remains zero. Since profits at $\underline{p}$ are zero, profits at $\overline{p}_1$ must be zero too. Note that residual demand for the high-priced seller gradually decreases when capacities grow further. This implies that the residual demand choke price is declining and therefore $\overline{p}_1$ declines as well. The range of better-response prices is narrowing until $k_1 \geqslant \frac{10}{9}$ and $k_2 \geqslant 1$. At that point, the MSS coincides with the pure-strategy Nash equilibrium in which both firms charge a price of zero.

The MSS can be compared to the mixed-strategy support $[\widehat{p}_1, p_1^*]$. A non-degenerate Nash equilibrium in mixed strategies exists when capacities are within the intermediate range $0.345 \approx \underline{k}_1 \leqslant k_1 < \frac{10}{9}$. In Figure 5, the mixed-strategy support is the vertical distance between the dashed lines. The upper bound of the support is given by the maximizer of the residual profit function π_1^h, which is linearly decreasing in k_1. Notice that for this range of capacities, the upper bound of the MSS, $\overline{p}_1$, is higher than p_1^*, because there are prices in excess of this maximizer that still constitute a better-response. Note further that the mixed-strategy support depends quadratically

[20]This maximum MSS price interval is reached at $k_1 = 1 - k_2$. Using $k_2 = \frac{9}{10}k_1$, this gives $k_1 \approx 0.53$.

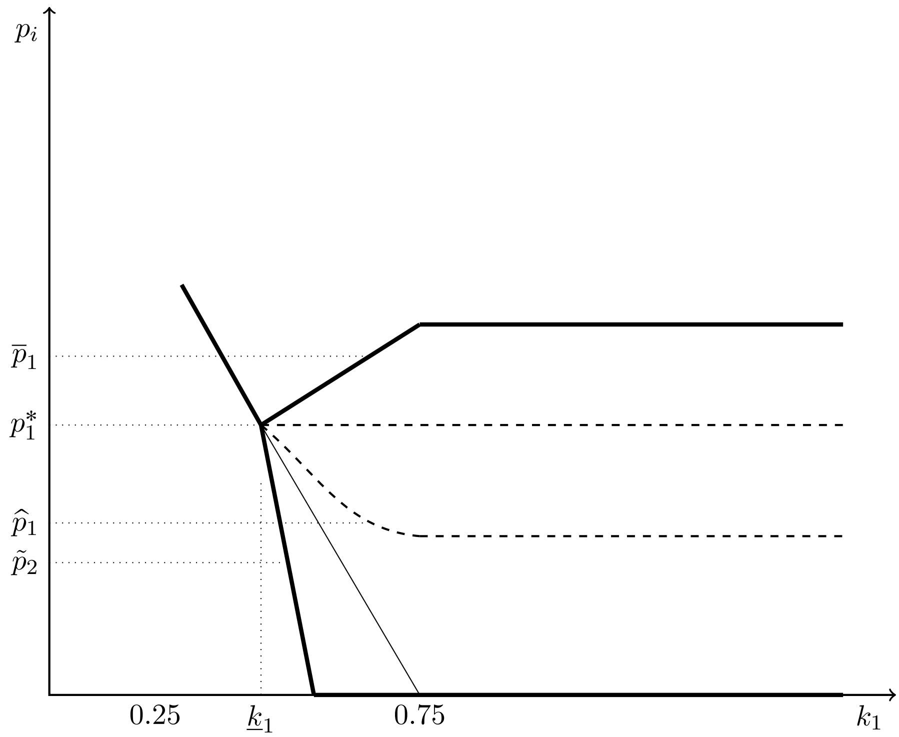

Figure 6: Myopic Stable Set in a linear demand duopoly with $k_2 = \frac{1}{4}$.

on k_1 and reaches its maximum at $k_1^* \approx 0.618$. Finally, observe that the lower bound of the mixed-strategy support, $\widehat{p}_1$, exceeds both $\underline{p}$ and $\widetilde{p}_2$. Figure 5 thus visualizes how the MSS strictly includes the support of the mixed-strategy Nash equilibrium (Theorem 3).

Figure 6 gives another illustration of the same linear demand duopoly example, but this time the capacity level of the smallest firm is kept fixed at $k_2 = \frac{1}{4}$. Starting at $k_1 = 0.25$, this means that the difference in firm size is growing when k_1 increases. The threshold value at which the existence of a pure-strategy equilibrium breaks down is at $\underline{k}_1 = \frac{(1-k_2)}{2} = 0.375$. At that point, the range of better-responses widens until $k_1 = 0.75$ for the same reasons as before. Yet, and unlike the scenario in Figure 5, $\overline{p}_1$ is not declining and remains constant for larger values of k_1. This is because k_2 is constant, which means that the residual demand for firm 1 when it is the higher priced seller is constant too. This, in turn, implies that the maximizer is independent of k_1 as well as the iso-profit price $\overline{p}_1$.

Similar to the case depicted in Figure 5, Figure 6 also visualizes how the mixed-strategy support (given by the vertical distance between the dashed lines) is strictly included in the MSS as indicated by the solid (thick) black line (Theorem 3). Furthermore, Figure 6 illustrates the positive relationship between the size of the MSS and the size of the biggest market player when capacities are in an intermediate range (Proposition 3).

6 Concluding Remarks

Within the growing body of work on behavioral industrial organization, there is an increasing focus on behavioral aspects of the firm. In this paper, we have relaxed the common assumption that firms are pure profit-maximizers and supposed that sellers seek myopic improvements instead. Under this assumption, we addressed a classic and persistent question in economics: How are prices determined in industries with a few powerful firms? To analyze this oligopoly pricing problem, we employed the Myopic Stable Set stability concept within the context of a capacity-constrained pricing game and established the existence of a unique MSS for any given level of capacities. This result was then compared with the standard Nash solution.

A main takeaway from our analysis is that the less demanding behavioral assumption of firms choosing myopic better-responses does not qualitatively affect existing Nash price predictions. If the set of pure-strategy Nash equilibria is nonempty, like when capacities are sufficiently large or small, it coincides with the MSS. With moderate-sized capacities, the Nash equilibrium is in mixed strategies. For these cases, all prices in the mixed-strategy support are part of the MSS. This set solution therefore offers an alternative foundation for oligopoly pricing. Moreover, we have shown that the MSS provides a rationale for different types of pricing dynamics. In particular, it gives an explanation for the emergence of Edgeworth-like price cycles as well as states of hyper-competition in which supply falls short of market demand.

We see several avenues for future research. One is to use the notion of MSS within the context of other oligopoly models. For example, Demuynck, Herings, Saulle and Seel (2019b) have recently characterized the MSS for a homogeneous-good Bertrand duopoly with asymmetric costs. A potentially interesting variation on this paper's capacity-constrained pricing model would be to assume that production precedes sales. Another avenue is to analyze oligopoly pricing under different behavioral assumptions such as heterogeneity in rationality or competition among quasi-myopic agents.[21] Finally, and especially because the MSS is rich enough to permit heterogeneous pricing, we can imagine it to serve as a foundation for further empirical and experimental work.

[21] See Dixon (2020) for a recent study of strategic firm behavior under the assumption of almost-maximization, *i.e.*, competition among almost-rational sellers who choose almost best-responses.

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Appendix: Proofs

Proof of Lemma 1. Suppose that $D(0) > K$ so that $\underline{p} > 0$. Since $D(\underline{p}) = K$ and demand is decreasing in price, it holds that $D(p_i) \geqslant K$ when $p_i \leqslant \underline{p}$. This means that $D(p_i) \geqslant k_i$ and $D(p_i) - K_{-i} \geqslant k_i$. It also implies that $\frac{k_i}{\sum_{j\in\Omega(p_i)} k_j}\left(D(p_i) - \sum_{j\in\Delta(p_i)} k_j\right) \geqslant k_i$, because $D(p_i) \geqslant K \geqslant \sum_{j\in\Omega(p_i)} k_j + \sum_{j\in\Delta(p_i)} k_j$. Hence, if $0 < p_i \leqslant \underline{p}$, then firm $i \in N$ always produces at capacity and therefore $\pi_i = p_i k_i$. ■

Proof of Proposition 1. If $K_{-1} \geqslant D(0)$, then $\underline{p} = 0$. To begin, suppose all firms set the same price. If all price at some $p' > \underline{p} = 0$, then none of them is capacity-constrained since $K > K_{-1} \geqslant D(0) > D(p')$. Hence, each seller has an incentive to (marginally) undercut his rivals. If all price at $\underline{p} = 0$, then firm 1 has no incentive to deviate since $K_{-1} \geqslant D(0)$. It would therefore face no residual demand at a price above zero. As the largest firm has no incentive to deviate, none of the firms has an incentive to deviate. We conclude that, when $K_{-1} \geqslant D(0)$, there is a symmetric pure-strategy Nash equilibrium with all firms pricing at $\underline{p} = 0$.

In addition, there are many asymmetric pure-strategy Nash equilibria, which have in common that there is a subset of sellers who price at zero. To see this, suppose that, by contrast, all set a price strictly above zero. Suppose further there is one firm charging the strictly highest price. In that case, this firm faces no demand since $K_{-1} \geqslant D(0)$. Hence, it would be better off by charging a lower price, *e.g.*, match the price of the lowest-priced firm(s).

Suppose then that there are two or more sellers who set the strictly highest price. If they face no residual demand, there is again an incentive to deviate, *e.g.*, they would be better off by matching the lowest price in the industry. Yet, if they do face residual demand, then they are not capacity-constrained. To see this, suppose the highest-priced sellers set a price $p' > 0$. They are then not capacity-constrained when:

$$\frac{k_i}{\sum_{j\in\Omega(p')} k_j}\left(D(p') - \sum_{j\in\Delta(p')} k_j\right) < k_i \iff D(p') < \sum_{j\in\Omega(p')} k_j + \sum_{j\in\Delta(p')} k_j = K,$$

which holds since $K > D(0) > D(p')$.

Next, note that in this case undercutting p' slightly is beneficial when:

$$(p'_i - \epsilon) \cdot min\{k_i, D(p'_i - \epsilon) - \sum_{j\in\Delta(p'_i-\epsilon)} k_j\} > p'_i \cdot \frac{k_i}{\sum_{j\in\Omega(p'_i)} k_j}\left(D(p'_i) - \sum_{j\in\Delta(p'_i)} k_j\right),$$

with $\epsilon > 0$ and sufficiently small. If the undercutting seller is capacity-constrained, then cutting its price to $p_i' - \epsilon$ is beneficial when:

$$
\begin{aligned}
(p_i' - \epsilon)k_i > p_i' \frac{k_i}{\sum_{j \in \Omega(p_i')} k_j} \left(D(p_i') - \sum_{j \in \Delta(p_i')} k_j \right) &\iff \\
p_i' - \epsilon > p_i' \frac{1}{\sum_{j \in \Omega(p_i')} k_j} \left(D(p_i') - \sum_{j \in \Delta(p_i')} k_j \right) &\iff \\
p_i' - p_i' \frac{1}{\sum_{j \in \Omega(p_i')} k_j} \left(D(p_i') - \sum_{j \in \Delta(p_i')} k_j \right) > \epsilon &\iff \\
p_i' \left[1 - \frac{1}{\sum_{j \in \Omega(p_i')} k_j} \left(D(p_i') - \sum_{j \in \Delta(p_i')} k_j \right) \right] > \epsilon.
\end{aligned}
$$

Note that the LHS is positive when the term inside the square brackets is positive. That is,

$$
1 > \frac{1}{\sum_{j \in \Omega(p_i')} k_j} \left(D(p_i') - \sum_{j \in \Delta(p_i')} k_j \right) \iff K = \sum_{j \in \Omega(p_i')} k_j + \sum_{j \in \Delta(p_i')} k_j > D(p_i'),
$$

which holds.

If the undercutting seller is not capacity-constrained, then cutting its price to $p_i' - \epsilon$ is beneficial when:

$$
\begin{aligned}
(p_i' - \epsilon) D(p_i' - \epsilon) > p_i' \left[\frac{k_i}{\sum_{j \in \Omega(p_i')} k_j} D(p_i') \right] &\iff \\
p_i' D(p_i' - \epsilon) - \epsilon D(p_i' - \epsilon) > p_i' \left[\frac{k_i}{\sum_{j \in \Omega(p_i')} k_j} D(p_i') \right] &\iff \\
p_i' D(p_i' - \epsilon) - p_i' \left[\frac{k_i}{\sum_{j \in \Omega(p_i')} k_j} D(p_i') \right] > \epsilon D(p_i' - \epsilon) &\iff \\
p_i' - \frac{k_i}{\sum_{j \in \Omega(p_i')} k_j} \frac{D(p_i')}{D(p_i' - \epsilon)} p_i' > \epsilon &\iff \\
p_i' \left[1 - \frac{k_i}{\sum_{j \in \Omega(p_i')} k_j} \frac{D(p_i')}{D(p_i' - \epsilon)} \right] > \epsilon.
\end{aligned}
$$

The LHS is strictly positive when:

$$
D(p_i' - \epsilon) > \frac{k_i}{\sum_{j \in \Omega(p_i')} k_j} D(p_i'),
$$

which holds because $D(p_i' - \epsilon) > D(p')$ and $k_i < \sum_{j \in \Omega(p_i')} k_j$. Hence, also in this case, each of the highest-priced sellers would have an incentive to (marginally) lower his price. We conclude that in equilibrium there is a subset of firms that price at zero.

For such a subset S to emerge in equilibrium it must hold that none of the firms $i \in S$ has an incentive to hike its price. Note that this is true for each subset that is *minimal*, *i.e.*, each coalition

S for which it holds that $\sum_{j \in S \setminus \{i\}} k_j \geqslant D(0)$, for all $i \in S$. Moreover, all sellers who are not part of such a minimal subset can charge any price since all prices give zero profits.

Taken together, therefore, the set of (a)symmetric pure-strategy Nash equilibria when $K_{-1} \geqslant D(0)$ is given by:

$$\text{NE} = \left\{ p \in P \middle| p \in \prod_{i \in S} \{0\} \times \prod_{i \in N \setminus S} [0, \infty) \right\}.$$

Now suppose that $K_{-1} < D(0)$. We can distinguish two cases: (1) $K \geqslant D(0)$ so that $\underline{p} = 0$, and (2) $K < D(0)$ so that $\underline{p} > 0$.

Case (1): If $K \geqslant D(0) > K_{-1}$, then $\underline{p} = 0$. We argue that there can be no pure-strategy Nash equilibrium in this case. To begin, note that there is no symmetric pure-strategy Nash equilibrium. If all firms would price at 0, then firm 1 would have an incentive to hike its price since $K_{-1} < D(0)$. If all sellers would price at some $p' > \underline{p} = 0$, then none of them is capacity-constrained since $K \geqslant D(0) > D(p')$. Consequently, each supplier has an incentive to (marginally) undercut its rivals. We conclude that there is no symmetric pure-strategy Nash equilibrium in this case.

Let us now argue that there does not exist an asymmetric pure-strategy Nash equilibrium either. First, notice that in such an equilibrium no firm can set a price at 0. Since $K_{-1} < D(0)$, this clearly holds for firm 1 as this firm always has a price above zero for which its (residual) demand is positive. However, given that firm 1 prices above zero, there is no combination of firms (other than firm 1) with a combined capacity sufficient to meet market demand at a price of zero. This implies that each firm that prices at zero has an incentive to hike its price. We conclude that there is no asymmetric pure-strategy Nash equilibrium in which one or more firms price at zero.

Suppose then that each firm prices above zero. In this case, there are one or more firms setting the strictly highest price. These sellers either face no demand in which case they have an incentive to set a lower price or they do face residual demand. By the same logic as above, however, none of them would be capacity-constrained and each of them has an incentive to (marginally) lower its price. We conclude that there is no (a)symmetric pure-strategy Nash equilibrium when $K \geqslant D(0) > K_{-1}$.

Case (2): If $K_{-1} < K < D(0)$, then $\underline{p} > 0$. We first argue that in this case there cannot be an asymmetric pure-strategy Nash equilibrium. If there was, then one or more firms would be charging the strictly highest price, say p'. If $0 < p' \leqslant \underline{p}$, then lower-priced sellers could profitably raise their price till p', because in this case it holds that $K \leqslant D(p')$. Suppose then that $p' > \underline{p}$ and that there are two or more firms charging the strictly highest price.

These sellers are not capacity-constrained since $K > D(p')$ when $p' > \underline{p}$. In this case, and as shown in the first part of this proof, each has an incentive to (marginally) lower its price.

This leaves the possibility of a single highest-priced firm. Following a similar logic as above, this firm cannot be capacity-constrained or have zero demand in equilibrium. In case it would be capacity-constrained, lower-priced firms could profitably raise their prices. In case of zero demand, the highest-priced seller could profitably deviate to a lower price. The only possibility is therefore that there is a single highest-priced seller with positive residual demand who is not producing at capacity. Suppose then that this highest-priced seller is pricing at $p' > \underline{p}$. This implies that lower-priced suppliers are capacity-constrained and therefore could raise their prices below, but arbitrarily close to p'. However, in that case, the highest-priced seller has an incentive to match the price of his rivals, because:

$$p'\frac{k_i}{K}D\left(p'\right) > p'\left(D\left(p'\right) - K_{-i}\right) \iff \frac{k_i}{K}D\left(p'\right) > D\left(p'\right) - K_{-i} \iff$$
$$K_{-i} > D\left(p'\right)\left[1 - \frac{k_i}{K}\right] \iff K_{-i} > D\left(p'\right)\left[\frac{K_{-i}}{K}\right] \iff K > D\left(p'\right),$$

which holds. We conclude that if there is a pure-strategy Nash equilibrium in this case, then it must be symmetric.

Suppose therefore that all firms charge the same price. If all price at $p' < \underline{p}$, then each firm can profitably deviate to a higher price (Lemma 1). In a similar vein, if all price at $p' > \underline{p}$, then $K > D\left(p'\right)$ so that each has an incentive to (marginally) undercut its rivals.

This leaves all firms pricing at $\underline{p}$ as the candidate equilibrium. Clearly, since all sellers produce at capacity in this case, none has an incentive to cut price. Moreover, if $K \leqslant D(p_1^*)$, then none of the firms has an incentive to hike its price. To see this, suppose that $K = D(p_1^*)$ so that $\underline{p} = p_1^*$. Since its first-order condition for a maximum is satisfied at $\underline{p} = p_1^*$, firm 1 does not have an incentive to hike its price:

$$\partial\pi_1^h(\underline{p})/\partial p_1 = D(\underline{p}) - K_{-1} + \underline{p}D'(\underline{p}) = D(p_1^*) - K_{-1} + p_1^* D'(p_1^*) = 0.$$

As for all other firms, $i \in N\backslash\{1\}$, note that $p_1^* \geqslant p_2^* \geqslant ... \geqslant p_n^*$ by strict concavity of the residual profit functions. Therefore, it holds that:

$$\partial\pi_i^h(\underline{p})/\partial p_i = D(\underline{p}) - K_{-i} + \underline{p}D'(\underline{p}) < 0,$$

which implies that none of the firms has an incentive to raise price. Finally, note that if $K < D(p_1^*)$, then $\underline{p} > p_1^*$ so that firm 1's marginal residual profit is negative at $\underline{p}$ and therefore also for its smaller capacity rivals. Hence, also in this case none of the firms has an incentive to hike its price. We conclude that there is a unique symmetric pure-strategy Nash equilibrium when $K \leqslant D(p_1^*)$ and it has all firms pricing at $\underline{p} > 0$. ■

Proof of Proposition 2. The proof is by construction. Following the definition of NE as provided in Section 4.1, we distinguish two cases and consider each case in turn.

Case (1): Suppose that $K_{-1} \geqslant D(0)$. To show that $f^{\infty}(p) \cap \text{NE} \neq \emptyset$ for each $p \notin NE$, we proceed in four steps.

Step 1: Following the proof of Proposition 1, if $K_{-1} \geqslant D(0)$, then there are two types of non-Nash price profiles: (i) a price profile with some firms pricing at zero, or (ii) a price profile with all firms pricing above zero. In case of (ii), move to Step 2. In case of (i), let S be the set of sellers who price at zero and let $p \in P \backslash NE$ be the corresponding price profile. Since the price profile p is not a Nash equilibrium, the largest member of S can profitably raise its price. Note that the resulting price profile is also not a Nash equilibrium so that we can repeat the argument. We conclude that there exists a sequence, which results in all firms charging a strictly positive price.

Step 2: Following Step 1, there exists a profitable price path from a non-Nash price profile with some firms pricing at zero to a non-Nash price profile with all firms pricing strictly above zero. Let $p' \in P \backslash NE$ be a non-Nash price profile with all firms charging a strictly positive price. We can again distinguish two cases: (i) all sellers set the same strictly positive price, or (ii) there are two or more firms charging a different strictly positive price. In case of (ii), move to Step 3. In case of (i), note that since $K > D(0)$ it is profitable for each firm to (marginally) undercut the price of its competitors. Hence, there is a profitable deviation in this case resulting in a price profile consisting of two or more different prices.

Step 3: Let $p'' \in P \backslash NE$ be a price profile resulting from Step 1 and Step 2. That is, p'' exclusively consists of prices above zero and contains at least two different prices. We can again distinguish two cases: (i) there are two or more firms charging the strictly highest price, or (ii) there is a single seller setting the strictly highest price. In case of (ii), move to Step 4. In case of (i), and following the proof of Proposition 1, the highest-priced firms are not capacity-constrained and can profitably undercut their highest-priced rivals.

Step 4: By steps 1,2 and 3, there is a sequence of myopic improvements from any non-Nash price profile to a non-Nash price profile with (i) strictly positive prices only, and (ii) a single strictly highest price. Let p be a given non-Nash price profile with these two characteristics and let the single highest-priced seller be denoted by h. Note that, since $K_{-1} \geqslant D(0)$, the highest-priced seller has no residual demand. Consequently, this firm can profitably deviate to a price p_h° lower than the lowest price in p and arbitrarily close to zero: $||p_h^{\circ} - 0|| < \epsilon$, for all $\epsilon > 0$. This would create a situation with a new highest-priced firm (perhaps via Step 3) and the argument can be repeated. This implies that there is a sequence of myopic improvements from the price profile p to a price profile with a sufficient number of sellers pricing arbitrarily close to zero. That is, there is a $\kappa > 0$ such that the κ-fold iteration of Step 4 generates a sequence:

$$p = p^0, p^1 \in f(p^0), p^2 \in f(p^1), ..., p^\kappa \in f(p^{\kappa-1}),$$

where p^κ is arbitrarily close to some $p' \in NE$: $||p^\kappa - p'|| < \epsilon$, for all $\epsilon > 0$. Then, by definition, $p' \in f^\infty(p)$ so that $f^\infty(p) \cap \text{NE} \neq \emptyset$.

Case (2): Now suppose that $K \leqslant D(p_1^*)$ so that all firms pricing at $\underline{p} > 0$ is the unique pure-strategy Nash equilibrium (Proposition 1). To begin, note that each seller who prices below $\underline{p}$ can profitably raise his price to $\underline{p}$ (Lemma 1). The remaining type of non-Nash price profile to consider is therefore one with all prices weakly above and at least one strictly above $\underline{p}$.

If there are two or more firms charging the strictly highest price (above $\underline{p}$), then the situation is comparable to Step 2 and Step 3 of Case (1) above. That is, a highest-priced seller can profitably deviate to a price (marginally) below the highest price. The resulting price profile is then one with a single strictly highest price. The maximum profit for this highest-priced firm (as before, indicated with subscript h) is obtained when it sets its residual profit-maximizing price, *i.e.*, the price p_h^* solving:

$$D(p_h^*) - \sum_{j \in \Delta(p_h^*)} k_j + p_h^* D'(p_h^*) = 0.$$

Note that the solution to this maximization problem would be no different when all its competitors price at $\underline{p}$. Since all firms pricing at $\underline{p}$ is a pure-strategy Nash equilibrium, however, it must hold that $\pi_h^h(p_h^*) \leqslant \pi_h(\underline{p})$. This implies that a single highest-priced seller can myopically improve by charging $\underline{p}$ and produce at capacity. This argument can be finitely repeated until all firms price at $\underline{p}$. We conclude that if $K \leqslant D(p_1^*)$, then $\underline{p} \in f^{\mathbb{N}}(p) \cap \text{NE} \neq \emptyset$, which implies $f^\infty(p) \cap \text{NE} \neq \emptyset$. ■

Proof of Theorem 1. The set NE as defined in Section 4.1 is a MSS when it is *closed* and satisfies *deterrence of external deviations*, *asymptotic external stability*, and *minimality*.

Closedness: If $K \leqslant D(p_1^*)$, then the pure-strategy Nash equilibrium is a singleton, which is closed. If $K_{-1} \geqslant D(0)$, then the set NE is given by:

$$NE = \left\{ p \in P \middle| p \in \prod_{i \in S} \{0\} \times \prod_{i \in N \setminus S} [0, \infty) \right\}.$$

Hence, it is effectively the product of a finite number of closed sets, which is closed.

Deterrence of External Deviations: Notice that the set of pure-strategy Nash equilibria is effectively the set of undominated strategy profiles: NE $= \{p \in P | f(p) = p\}$, which implies that no firm can profitably deviate to a price profile outside NE.

Asymptotic External Stability: This condition holds by Proposition 2, which establishes that the capacity-constrained pricing game exhibits the weak improvement property. Hence, from any price profile not in NE it is possible to get arbitrarily close to a pure-strategy equilibrium by a finite number of myopic improvements.

Minimality: Since the set NE is closed and the previous two conditions hold, minimality follows directly from Corollary 3.11 in Demuynck, Herings, Saulle and Seel (2019a); a mirror result which effectively shows that MSS $\supseteq$ NE when the set of pure-strategy Nash equilibria is nonempty.

Combining the above, we conclude that the set NE is a MSS. It remains to be shown that it is also the unique MSS.

Uniqueness: Suppose there would be another MSS, M, different from NE. As NE is a MSS, first note that neither $M \supset$ NE, nor NE $\supset M$, because otherwise the minimality requirement would be violated for either M or NE. Moreover, note that neither $M \cap$ NE $= \emptyset$, nor $M \cap$ NE $\neq \emptyset$ with $M \neq$ NE. If so, then there would be a price profile in NE that is not in M. Yet, for each price profile in NE it holds that no firm has a profitable deviation to a price profile outside NE, which implies that the Asymptotic External Stability condition would be violated for M. We conclude that $M =$ NE and therefore that NE is the unique MSS. ■

Proof of Lemma 2. Let us prove each of the three cases in turn.

(i) For each $i \in N$, if $D(0) > K_{-i} > D(p_i^*) - k_i$, then $D(\underline{p}) > D(p_i^*)$ because either (1) $K < D(0)$ and $\underline{p} > 0$ so that $D(\underline{p}) = K > D(p_i^*)$, or (2) $K \geqslant D(0)$ and $\underline{p} = 0$ so that $D(\underline{p}) = D(0) > D(p_i^*)$. Hence, in this case, $p_i^* > \underline{p} \geqslant 0$. Since the residual profit functions are strictly concave and have a unique maximizer at p_i^*, it follows that the iso-profit price is at the decreasing part of the residual profit function: $\overline{p}_i > p_i^* > 0$, for all $i \in N$.

Let us now show that the iso-profit price is increasing with firm capacity. To that end, consider two firms i and j with $k_i > k_j$ and suppose that $\underline{p} > 0$. Suppose further that they both pick a price p from $\left(0, D^{-1}(K_{-i})\right)$. Comparing their residual profits gives:

$$\begin{aligned} \pi_i^h(p) - \pi_j^h(p) &= p\,(D(p) - K_{-i}) - p\,(D(p) - K_{-j}) \\ &= p\,(K_{-j} - K_{-i}) = p\,(k_i - k_j) > 0. \end{aligned} \tag{5}$$

Moreover, for $\underline{p} > 0$ and following Definition 5 of the iso-profit price, it must hold that:

$$\pi_i^h(\overline{p}_i) - \pi_j^h\left(\overline{p}_j\right) = \underline{p}\,(k_i - k_j) > 0. \tag{6}$$

To show that $\overline{p}_i > \overline{p}_j > 0$, suppose the opposite, *i.e.*, $\overline{p}_j > \overline{p}_i$, in view of a contradiction. As established above, firms' residual profits are decreasing at their iso-profit price so that:

$$\pi_i^h\left(\overline{p}_j\right) < \pi_i^h(\overline{p}_i) \text{ for } \overline{p}_j > \overline{p}_i,$$

and, therefore:

$$\pi_i^h\left(\overline{p}_j\right) - \pi_j^h\left(\overline{p}_j\right) < \pi_i^h(\overline{p}_i) - \pi_j^h\left(\overline{p}_j\right) = \underline{p}\,(k_i - k_j)\,.$$

Notice, however, that:

$$\begin{aligned} \pi_i^h\left(\overline{p}_j\right) - \pi_j^h\left(\overline{p}_j\right) &= \overline{p}_j\left(D(\overline{p}_j) - K_{-i}\right) - \overline{p}_j\left(D(\overline{p}_j) - K_{-j}\right) \\ &= -\overline{p}_j\,(K - k_i) + \overline{p}_j\,(K - k_j) = \overline{p}_j\,(k_i - k_j) > \underline{p}\,(k_i - k_j)\,, \end{aligned}$$

which contradicts the previous result that $\pi_i^h\left(\overline{p}_j\right) - \pi_j^h\left(\overline{p}_j\right) < \underline{p}\,(k_i - k_j)$.

Now suppose $\underline{p} = 0$. In this case, and again following Definition 5, it holds that:

$$\pi_i^h(\overline{p}_i) = \overline{p}_i\,(D(\overline{p}_i) - K_{-i}) = \underline{p} \cdot k_i = 0,$$

which implies that the iso-profit price of firm i is given by:

$$\overline{p}_i = D^{-1}(K_{-i}) > 0,$$

and, therefore:

$$D^{-1}(K_{-i}) > D^{-1}(K_{-j}) > 0,$$

for every $i, j \in N$ with $k_i > k_j$. We conclude that if $k_i > k_j$, then $\overline{p}_i > \overline{p}_j > 0$, for all $i, j \in N$. Notice that this also shows that firm 1 has the highest iso-profit price.

(ii) Suppose now that $D(p_i^*) - K_{-i} \geqslant k_i$, for all $i \in N \backslash \{1\}$. Since in this case $D(p_i^*) \geqslant K$, it follows that $D(p_i^*) \geqslant K \equiv D(\underline{p})$ for every $i \neq 1$, which implies that $\underline{p} \geqslant p_i^*$. Since in this case every firms i (except firm 1) sells at capacity (see Lemma 1), its profit line $\underline{p}k_i$ intersects the residual profit π_i^h either at the maximum or at the decreasing part. Consequently, the only price for which $\underline{p} \cdot k_i = \pi_i^h(p_i)$ is the market-clearing price $\underline{p}$ at which the residual profit of firm i reaches its peak and declines afterward at any higher price $p_i > \underline{p}$. Finally, recall that in case of intermediate capacities: $D(p_1^*) < K$ and $D(0) > K_{-1}$. Hence, for firm 1 it always holds that $\overline{p}_1 > p_1^* > \underline{p}$.

(iii) The third situation to consider is when $D(0) \leqslant K_{-m}$, with $n \geqslant m > 1$. In this case, it holds that $D(0) < K$ and therefore $\underline{p} = 0$. Notice that residual demand for firm m is zero at all prices, which implies that all firms weakly smaller than firm m also face no residual demand. We conclude that $\overline{p}_i = \underline{p} = 0$ for each firm $i = m, m+1, m+2, ..., n$ with capacity $k_i \leqslant k_m$.

■

Proof of Lemma 3. Let us discuss the three cases in turn.

(i) Suppose that $K < D(0)$ and that firm 1 is the strictly largest seller. In this case, $\underline{p} > 0$ and the iso-profit price $\overline{p}_1$ is the price solving the following equation:

$$\pi_1^h(\overline{p}_1) = \overline{p}_1 \left(D(\overline{p}_1) - K_{-1}\right) = \underline{p} \cdot k_1.$$

The hyper-competitive prices are given by:

$$\pi_i^h(\overline{p}_1) = \overline{p}_1 \left(D(\overline{p}_1) - K_{-i}\right) = \tilde{p}_i \cdot k_i.$$

Hence, it immediately follows that $\widetilde{p}_1 = \underline{p}$. As to a firm $i \in N \backslash \{1\}$, note that

$$\underline{p} = \frac{\overline{p}_1 \left(D(\overline{p}_1) - K_{-1}\right)}{k_1} \text{ and } \widetilde{p}_i = \frac{\overline{p}_1 \left(D(\overline{p}_1) - K_{-i}\right)}{k_i}.$$

Comparing and rearranging gives:

$$\underline{p} - \widetilde{p}_i = \frac{\overline{p}_1 \left(D(\overline{p}_1) - K_{-1}\right)}{k_1} - \frac{\overline{p}_1 \left(D(\overline{p}_1) - K_{-i}\right)}{k_i} = \overline{p}_1 \cdot \frac{(k_1 - k_i)\left[K - D(\overline{p}_1)\right]}{k_1 k_i} > 0,$$

which holds because $\overline{p}_1 > \underline{p}$ and firm 1 is the strictly largest seller. We conclude that if $K < D(0)$ and each firm $i \in N \backslash \{1\}$ is strictly smaller than firm 1, then $0 < \widetilde{p}_i < \underline{p}$ and $\widetilde{p}_1 = \underline{p}$.

(ii) Given that $D(0) > K$ and following the same steps as under (i) above, it holds that

$$\widetilde{p}_i - \widetilde{p}_j = \overline{p}_1 \frac{(k_i - k_j)\left[K - D(\overline{p}_1)\right]}{k_i k_j} > 0,$$

for any two firms $i, j \in N$ with $k_i > k_j$.

(iii) If $D(0) \leqslant K_{-i}$, then $D(0) \leqslant K$ and therefore $\underline{p} = 0$. Moreover, the residual demand of firm $i \in N \backslash \{1\}$ at $\overline{p}_1$ is $D(\overline{p}_1) - K_{-i} \leqslant 0$, which implies $\pi_i^h(\overline{p}_1) = 0$ and therefore $\widetilde{p}_i = \underline{p} = 0$.

■

Proof of Lemma 4.

(i): Suppose $p_i = \overline{p}_1$. We show that for all $p_i \in P_i$ such that $\underline{p} < p_i < \overline{p}_1$ it holds that $\pi_i(p_i, p_{-i}) = p_i k_i$. The proof relies on the following claim:

Claim 1: For all $i \neq 1$ such that $p_i \in (\underline{p}, \overline{p}_1)$ then firm i is capacity-constrained.

Proof. Since $p_1 = \overline{p}_1$ then either, (i) $\underline{p} = 0$, which implies $D(\overline{p}_1) - \sum_{j \in \Delta(\overline{p}_1)} k_j = 0$, or (ii) $\underline{p} > 0$, which implies $D(\overline{p}_1) - \sum_{j \in \Delta(\overline{p}_1)} k_j > 0$. Taken together, this means:

$$D(\overline{p}_1) - \sum_{j \in \Delta(\overline{p}_1)} k_j \geqslant 0,$$

The above can be rewritten as

$$D(\overline{p}_1) - \sum_{j \in \Delta(\overline{p}_1) \backslash \{i\}} k_j \geqslant k_i, \text{ for each } i \in N \backslash \{1\}.$$

Consider some firm $i \in N \backslash \{1\}$ that prices at $p_i \in (\underline{p}, \overline{p}_1)$. Its demand is then generally given by:

$$D_i(p_i, p_{-i}) = \max \left\{ \frac{k_i}{\sum_{j \in \Omega(p_i)} k_j} \left(D(p_i) - \sum_{j \in \Delta(p_i)} k_j \right), 0 \right\}.$$

Note that since

$$D(\overline{p}_1) - \sum_{j \in \Delta(\overline{p}_1)} k_j \geqslant 0,$$

it holds that

$$D(p_i) - \sum_{j \in \Delta(p_i)} k_j > 0,$$

since $D(p_i) > D(\overline{p}_1)$ and $\sum_{j \in \Delta(p_i)} k_j < \sum_{j \in \Delta(\overline{p}_1)} k_j$ for $p_i < \overline{p}_1$. Hence, each firm $i \in N \backslash \{1\}$ faces strictly positive demand when pricing below $\overline{p}_1$. Note further that this also implies that this firm is capacity-constrained when no other firm prices at p_i, because

$$D(p_i) - \sum_{j \in \Delta(p_i)} k_j > D(\overline{p}_1) - \sum_{j \in \Delta(\overline{p}_1) \backslash \{i\}} k_j \geqslant k_i.$$

because $D(p_i) > D(\overline{p}_1)$ and $\sum_{j \in \Delta(p_i)} k_j \leqslant \sum_{j \in \Delta(\overline{p}_1) \setminus \{i\}} k_j$ for $p_i < \overline{p}_1$.

Finally, suppose that there is at least one other firm pricing at $p_i \in (\underline{p}, \overline{p}_1)$. In this case, firm i is also capacity-constrained, because

$$
\begin{aligned}
\frac{k_i}{\sum_{j \in \Omega(p_i)} k_j} \left(D(p_i) - \sum_{j \in \Delta(p_i)} k_j \right) > k_i \iff \\
D(p_i) - \sum_{j \in \Delta(p_i)} k_j > \sum_{j \in \Omega(p_i) \setminus \{i\}} k_j + k_i \iff \\
D(p_i) - \sum_{j \in \Delta(p_i)} k_j - \sum_{j \in \Omega(p_i) \setminus \{i\}} k_j > k_i.
\end{aligned}
$$

This inequality holds, because $D(p_i) > D(\overline{p}_1)$ and $\sum_{j \in \Delta(p_i)} k_j + \sum_{j \in \Omega(p_i) \setminus \{i\}} k_j \leqslant \sum_{j \in \Delta(\overline{p}_1)} k_j$. □

Since Claim 1 holds, we conclude that if p_i is such that $\underline{p} \leqslant p_i < \overline{p}_1$, then $\pi_i(p_i, p_{-i}) = p_i k_i$ which is increasing in price. Hence, $\pi_i(p_i, p_{-i}) > \pi_i(\underline{p})$ when $p_i \in (\underline{p}, \overline{p}_1)$ and $p_1 = \overline{p}_1$, for each firm $i \in N \setminus \{1\}$.

(ii): By definition, $\tilde{p}_i = \min\{p_i \in P_i | \pi_i(p_i) = \pi_i^h(\overline{p}_1)\}$. By Lemma 1, we know that if $0 < p_i \leqslant \underline{p}$, then $\pi_i = p_i k_i$. Consequently, since π_i is strictly increasing in p_i in the range $(\tilde{p}_i, \underline{p}]$ it holds that $\pi_i(p_i, p_{-i}) > \pi_i^h(\overline{p}_1)$ for $p_i \in (\tilde{p}_i, \underline{p}]$ ■

The following Lemma 5 and Lemma 6 are used in the proof of Theorem 2.

Lemma 5. *If $p \in M$ and $p' \in f^{\infty}(p)$, then $p' \in M$.*

Proof of Lemma 5. Towards a contradiction, suppose that $p' \notin M$ when $p' \in f^{\infty}(p)$ and $p \in M$. Since M is closed, there is a $\delta > 0$ such that $\mathcal{B}_\delta(p') \cap M = \varnothing$, where $\mathcal{B}_\delta$ is the open ball with radius δ. Furthermore, by the definition of f^{∞}, there is a $\kappa \in \mathbb{N}$ and a $p^\kappa \in f^\kappa(p)$ such that $p^\kappa \in \mathcal{B}_\delta(p')$, which means $p^\kappa \notin M$. Since $p^\kappa \in f^\kappa(p)$, there is a sequence $p^0, p^1, ..., p^\kappa$ of length κ such that

$$p^0 = p, p^1 \in f(p^0), ..., p^\kappa \in f(p^{\kappa-1}).$$

Let $\kappa' \in \{1, ..., \kappa\}$ be such that $p^{\kappa'}$ is the first element in this sequence with the property that $p^{\kappa'} \notin M$. Hence, $p^{\kappa'-1} \in M$ and $p^{\kappa'} \in f(p^{\kappa'-1})$, which violates deterrence of external deviations of M. Consequently, $p' \in M$. ■

Lemma 6. *Let $\pi_i^s(p_i)$ denote the profit of firm $i \in N$ when at least one other firm $j \neq i$ is charging the same price p_i. Then, it holds that $\pi_i^s(p_i) > \pi_i^h(p_i)$ and $\pi_i^\ell(p_i) > \pi_i^h(p_i)$ for any $p_i \in (\underline{p}, \alpha)$.*

Proof of Lemma 6. Let us start by showing that:

$$\pi_i^s(p_i) > \pi_i^h(p_i).$$

We can distinguish two cases: (1) the firms charging the same price are capacity-constrained, or (2) the firms charging the same price are not capacity-constrained. Note that in either case the single highest-priced firm is not capacity-constrained, because with $p_i \in (\underline{p}, \alpha)$ it holds that $K > D(p_i)$. As to (1), we have that

$$\pi_i^s(p_i) > \pi_i^h(p_i) \Longleftrightarrow p_i k_i > p_i \left[D(p_i) - \sum_{j \in \Delta(p_i)} k_j \right] \Longleftrightarrow K > D(p_i),$$

which holds. As to (2), we have that

$$\pi_i^s(p_i) > \pi_i^h(p_i) \Longleftrightarrow p_i \frac{k_i}{\sum_{j \in \Omega(p_i)} k_j} \left[D(p_i) - \sum_{j \in \Delta(p_i)} k_j \right] > p_i \left[D(p_i) - \sum_{j \in \Delta'(p_i)} k_j \right],$$

with $\Delta'(p_i) \neq \Delta(p_i)$. This is equivalent to

$$\frac{k_i}{\sum_{j \in \Omega(p_i)} k_j} \left[D(p_i) - \sum_{j \in \Delta(p_i)} k_j - \sum_{j \in \Omega(p_i)} k_j + \sum_{j \in \Omega(p_i)} k_j \right] > D(p_i) - \sum_{j \in \Delta'(p_i)} k_j,$$

or

$$\frac{k_i}{\sum_{j \in \Omega(p_i)} k_j} \left[D(p_i) - K + \sum_{j \in \Omega(p_i)} k_j \right] > D(p_i) - \sum_{j \in \Delta'(p_i)} k_j.$$

Rearranging gives

$$k_i \left[\frac{D(p_i) - K}{\sum_{j \in \Omega(p_i)} k_j} \right] > D(p_i) - \sum_{j \in \Delta'(p_i)} k_j - k_i \Longleftrightarrow k_i \left[\frac{D(p_i) - K}{\sum_{j \in \Omega(p_i)} k_j} \right] > D(p_i) - K.$$

Since $D(p_i) < K$, the above simplifies to

$$\frac{k_i}{\sum_{j \in \Omega(p_i)} k_j} < 1,$$

which holds. We conclude that $\pi_i^s(p_i) > \pi_i^h(p_i)$ when $p_i \in (\underline{p}, \alpha)$.

Let us now show that:

$$\pi_i^\ell(p_i) > \pi_i^h(p_i).$$

We can again distinguish two cases: (1) the firm charging the lowest price is capacity-constrained, or (2) the firm charging the lowest price is not capacity-constrained. As to (1), the story is the same as above. As to (2), we have that

$$\pi_i^\ell(p_i) > \pi_i^h(p_i) \Longleftrightarrow p_i D(p_i) > p_i \left[D(p_i) - \sum_{j \in \Delta(p_i)} k_j \right],$$

which holds because $\sum_{j\in\Delta(p_i)} k_j > 0$. We conclude that $\pi_i^{\ell}(p_i) > \pi_i^{h}(p_i)$ when $p_i \in (\underline{p}, \alpha)$. ■

Proof of Theorem 2. First note that the set M is closed by definition. In the following, we show that the set M also satisfies *Deterrence of External Deviations*, *Asymptotic External Stability* and *Minimality*. Finally, we prove that M is unique.

Deterrence of External Deviations: Let $p \in M$ be some price profile in M. We show that there is no profitable deviation to a price profile in $P \backslash M$. Take any firm $i \in N$ and suppose that it is the highest priced firm in the market. We have that either $D(0) > K_{-i}$ or $D(0) \leqslant K_{-i}$. In the former case, firm i has positive residual demand. Then, by strict concavity of π_i^h, its profit is lowest in M at $\overline{p}_1$. By Lemma 1 and Lemma 6, since $\pi_i^{\ell}(p_i) \geqslant \pi_i^h(p_i)$ and $\pi^s(p_i) \geqslant \pi_i^h(p_i)$ for all $p_i \in [\widetilde{p}_i, \overline{p}_1]$ there is no other price in the set M giving a lower profit. By definition of $\widetilde{p}_i$, such a profit $\pi_i^h(\overline{p}_1)$ is equivalent to the situation in which firm i charges $\widetilde{p}_i$. We therefore conclude that, whenever $D(0) > K_{-i}$, the lowest profit for any firm i is obtained either at $\widetilde{p}_i$ or at $\overline{p}_1$ given that it is the highest priced firm in the market. Note that if a firm i unilaterally deviates to a $p' \in P \backslash M$, it must be either that (a) $p_i' < \widetilde{p}_i$, or (b) $p_i' > \overline{p}_1$. Consider first case (a). Then, by Lemma 1, this firm will obtain $\pi_i(p_i') \equiv p_i' \cdot k_i < \pi_i(\widetilde{p}_i) \equiv \widetilde{p}_i \cdot k_i$ and therefore decreasing price below $\widetilde{p}_i$ is not an improvement. Take now case (b). If firm i deviates with a $p_i' > \overline{p}_1$, it holds that $\pi_i^h(p_i') < \pi_i^h(\overline{p}_1)$ and therefore for any price $p_i' > \overline{p}_1$ such deviation is unprofitable.

Finally, consider the case $D(0) \leqslant K_{-i}$. By Lemma 3, we have that $\widetilde{p}_i = 0$. Then, the only possible deviation to $P \backslash M$ for firm i is to some $p_i' > \overline{p}_1$. However, profits are zero at all prices in excess of $\overline{p}_1$ so that no firm has a profitable deviation to such a price.

Asymptotic External Stability: Consider a price profile $p \in P \backslash M$. We show that there exists a $p' \in M$ such that $p' \in f^{\mathbb{N}}(p)$. To begin, notice that if $p \in P \backslash M$, then there is at least one firm pricing below its hyper-competitive price $\widetilde{p}_i$ or above firm 1's iso-profit price $\overline{p}_1$.

Let $L(\widetilde{p}_i) = \{i \in N | p_i < \widetilde{p}_i\}$ be the set of sellers who are pricing below their hyper-competitive price and let $H(\overline{p}_1) = \{i \in N | p_i > \overline{p}_1\}$ be the set of sellers who price above firm 1's iso-profit price. Moreover, let $\lambda \geqslant 0$ and $\eta \geqslant 0$ denote the cardinality of $L(\widetilde{p}_i)$ and $H(\overline{p}_1)$, respectively.

Step 1: If $L(\widetilde{p}_i) = \varnothing$, then $H(\overline{p}_1) \neq \varnothing$; In this case, proceed with Step 2. If $L(\widetilde{p}_i) \neq \varnothing$ then, for each firm $i \in L(\widetilde{p}_i)$, $p_i < \widetilde{p}_i \leqslant \underline{p}$, so that $\pi_i(p_i, p_{-i}) = p_i k_i$ by Lemma 1. This implies that each firm $i \in L(\widetilde{p}_i)$ can profitably deviate to the market-clearing price $\underline{p} \in M$, which induces a sequence of price profiles:

$$p = p^0,\ p^1 \in f\left(p^0\right),\ p^2 \in f\left(p^1\right), \ldots, p^{\lambda} \in f\left(p^{\lambda-1}\right).$$

If $p^{\lambda} \in M$, then the *Asymptotic External Stability* condition is met. If $p^{\lambda} \notin M$, then proceed with Step 2.

Step 2: Let p^λ, with $\lambda \geqslant 0$, be the price profile resulting from Step 1. By construction, it holds that $L(\widetilde{p}_i) = \varnothing$ and $H(\overline{p}_1) \neq \varnothing$. Suppose $D(0) > K$ which implies that $\underline{p} > 0$. First, recall that $\widetilde{p}_i$ is the price solving $\pi_i(\widetilde{p}_i) = \pi_i^h(\overline{p}_1)$, for all $i \in N$. This implies $\pi_i^h(p_i) \leqslant \pi_i^h(\overline{p}_1)$ when $p_i \geqslant \overline{p}_1$, because $\pi_i^h(p_i)$ is strictly concave. Next, denote by $h(p^\lambda) \in H(\overline{p}_1)$ the firm charging the highest price at p^λ. Since $p_h^\lambda > \overline{p}_1$ by construction and following the previous observations in combination with Lemma 1, firm $h(p^\lambda)$ has a profitable deviation to $\underline{p}$. This induces a new price profile $p^{\lambda+1}$ in which case either all firms price weakly below $\overline{p}_1$, or there are still one or more firms pricing above $\overline{p}_1$. In case of the former, the *Asymptotic External Stability* condition is met, whereas in case of the latter we can repeat the argument. Denote by $h(p^{\lambda+1})$ the firm charging the highest price at $p^{\lambda+1}$. This firm has a profitable deviation to $\underline{p} > \widetilde{p}_{h(p^{\lambda+1})}$. Extending the above logic delivers a sequence:

$$p^{\lambda+1} \in f\left(p^\lambda\right),\ p^{\lambda+2} \in f\left(p^{\lambda+1}\right), ..., p^{\lambda+\eta} \in f\left(p^{\lambda+\eta-1}\right).$$

Hence, by construction, $p^{\lambda+\eta} = p' \in M$, and therefore the *Asymptotic External Stability* condition is met.

Finally, suppose that $D(0) \leqslant K$ so that $\underline{p} = 0$. Consequently, $K_{-1} < D(0)$ for otherwise all firms pricing at zero would constitute a pure-strategy Nash equilibrium. Now consider some price profile $p \notin M$ with at least one firm pricing above $\overline{p}_1$. If p is such that firm 1 prices at zero, then let this firm raise its price to p_1^*, which is profitable and in the set M, because $p_1^* < \overline{p}_1$. Note that since $K_{-1} - k_i < D(0)$ for any firm i other than firm 1, each firm pricing at zero can profitably raise its price to p_1^*. This results in a price profile with all firms strictly pricing above zero.

Next, consider the highest price in the market. If there are two or more firms charging the highest price, say p_i, then they are not capacity-constrained since $D(p_i) - \sum_{j\in\Delta(p_i)} k_j < \sum_{j\in\Omega(p_i)} k_j$, which is equivalent to $K > D(p_i)$ and this holds, because $K \geqslant D(0)$. If their profits are positive, then there is a myopic improvement by cutting their price slightly since this gives a discrete increase in demand. This yields a situation in which one firm charges the strictly highest price. As $D(\overline{p}_1) - K_{-1} = 0$ in this case, we have that $D(\overline{p}_1) - K_{-i} \leqslant 0$ for each firm i other than 1 and, therefore, $D(p_i') - K_{-i} < 0$ for any $p_i' > \overline{p}_1$ and $i \in N$. Hence, this single highest-priced firm receives zero profits and, hence, has a profitable deviation to p_1^*.

Minimality: Towards a contradiction, suppose that there exists a closed set $M' \subsetneq M$ satisfying *Deterrence of External Deviations* and *Asymptotic External Stability*. We distinguish two cases: either the *market-clearing price profile* $\underline{p} \in M'$, or the *market-clearing price profile* $\underline{p} \notin M'$.

Case 1: Suppose that $\underline{p} \in M'$. Note that at $\underline{p}$, if a firm i has a positive residual demand, then it has a profitable deviation to any p_i with $\underline{p} < p_i < \overline{p}_i$, by concavity of the residual profit functions. Note that this always holds for firm 1 (Lemma 2). Recall also that, by Lemma 2 we have that $\overline{p}_1 \geqslant \ldots \geqslant \overline{p}_n$. It follows that, from $\underline{p}$, the largest price interval is effectively determined by firm 1. Fix such a firm 1 and let it deviate to any p_1 for which $\underline{p} < p_1 < \overline{p}_1$. Thus, by the property of *deterrence of external deviations* of M' and the fact that $\underline{p} \in M'$, the following price profiles are contained in M':

$$M'_1 = \{p \in P | \underline{p} \leqslant p_1 < \overline{p}_1, p_j = \underline{p}, \forall j \neq 1\} \subseteq M'.$$

Moreover, as firm 1 can charge a price in M' arbitrarily close to $\overline{p}_1$, then, by Lemma 5, the following price profiles are also contained in M':

$$M'_1 \subset M'_2 = \{p \in P | \underline{p} \leqslant p_1 \leqslant \overline{p}_1, p_j = \underline{p}, \forall j \neq 1\} \subseteq M'.$$

Now fix $p_1 = \overline{p}_1$ in M'_2. Then, by Lemma 4, each firm i other than firm 1 has a profitable deviation to any p_i for which it holds that $\underline{p} < p_i < \overline{p}_1$. It then follows from the *deterrence of external deviations* of M' that

$$M'_2 \subset M'_3 = \{p \in P | \underline{p} \leqslant p_i \leqslant \overline{p}_1, \forall i \in N\} \subseteq M'.$$

Next, fix some $p \in M'_3$ with $p_i = \overline{p}_1$ for some $i \neq 1$ and $p_1 < \overline{p}_1$. In this case, it is implied by Lemma 4 (*ii*) that this firm i has a profitable deviation to some price $p'_i \in (\tilde{p}_i, \underline{p}]$. Note that, since the choice of i is arbitrary and the fact that M' satisfies the *deterrence of external deviations* condition, it follows that

$$M'_3 \subset M'_4 = \{p \in P | \tilde{p}_i < p_i \leqslant \overline{p}_1, \forall i \in N\} \subseteq M'.$$

Finally, note that since firm i can charge a price in M' arbitrarily close to $\tilde{p}_i$, it must hold by Lemma 5 that:

$$M'_4 \subset M'_5 = \{p \in P | \tilde{p}_i \leqslant p_i \leqslant \overline{p}_1, \forall i \in N\} \subseteq M',$$

and therefore $M'_5 = M \subseteq M'$, a contradiction.

Case 2: Suppose now that the price profile $\underline{p} \notin M'$. Fix some $p \in M'$. We show that $\underline{p} \in f^{\infty}(p)$. It then follows from Lemma 5 that $\underline{p} \in M'$, a contradiction.

Step 1: Let $L(p) = \{i \in N | p_i < \underline{p}\}$ be the set of sellers who are pricing below the market clearing price. Either $L(p)$ is empty or not. In case of the former proceed with Step 2. In case of the latter, note that each firm pricing below the market-clearing price has a profitable deviation to $\underline{p}$ (Lemma 1). By letting each firm to deviate we induce a new price profile $p' \in f^{\lambda}(p)$ for some $\lambda > 0$, in which each firm prices weakly above the market-clearing price.

Notice that, since M' satisfies *deterrence of external deviations*, it must hold that $p' \in M'$. If $p' = \underline{p}$, then we have a contradiction concluding the proof. Otherwise proceed with *Step 2*.

Step 2: Let p be the price profile resulting from Step 1. Thus p is such that all firms weakly price above $\underline{p}$ with at least one seller pricing strictly above $\underline{p}$. In this second step, we show there exists a path of myopic improvements such that the resulting price profile consists of two prices. Specifically, this price profile has one or more firms charging the highest price p_h and one or more firms charging the lowest price $p_h - \varepsilon$, with ε positive and sufficiently small.

To begin, consider a lowest priced firm l. If this firm is not capacity-constrained, then all higher priced sellers face zero residual demand. If $p_l = \underline{p}$, then all higher priced sellers can profitably match p_l in turn. This, induces a new price profile $\underline{p} \in f^{\mu}(p)$ for some $\mu > 0$, By the μ-fold iteration of *deterrence of external deviations* we have that $\underline{p} \in M'$, a contradiction concluding the proof. If $p_l > \underline{p}$, then each higher priced firm can profitably deviate to a price $p_l - \epsilon \geqslant \underline{p}$ for some $\epsilon > 0$. This results in a price profile with two prices: p_l and $p_l - \epsilon$.

Next, suppose that there is a single lowest priced seller who is capacity-constrained. In this case, we can distinguish two scenarios. Either, (i) it raises its price till p_l' for which $k_l = D\left(p_l'\right)$. That is, to the lowest price for which its capacity is non-binding, while remaining the lowest priced firm in the market, or (ii) it raises its price till it matches the price of the second lowest priced firm(s). We study the two cases separately.

Case (i): In this case, we are back in the first situation where none of the higher priced firms faces residual demand. Hence, they can profitably lower their price to $p_l' - \varepsilon$. Again, the result is a price profile with two prices: p_l' and $p_l' - \varepsilon$.

Case (ii): Let firm i be one of the lowest priced firms. Either, (iia) firm i raises its price until it is no longer capacity-constrained, or (iib) firm i matches the next lowest prices firm. We consider the two possibilities separately.

- **(iia):** Firm i raises its price till p_l'' for which it holds that $D\left(p_l''\right) - \sum_{j \in \Delta\left(p_l''\right)} k_j = k_i$. This implies that all firms pricing below p_l'' can also profitably raise their price till p_l'', whereas all firms pricing above p_l'' face no residual demand. Hence, these higher priced firms can myopically improve by lowering their price to $p_l'' - \epsilon$, which again results in a price profile with two prices: p_l'' and $p_l'' - \epsilon$.
- **(iib):** Firm i raises its price to the price of the next lowest priced firm(s) in which case all lower priced firms can do the same since they are capacity-constrained. This brings us back either to the situation described under (iia) above or iterate (iib) until the highest priced firms still face residual demand, in which case lower priced firms can raise their price till $p_h - \epsilon$, where p_h is the highest price in the market.

Step 3: Let $p \in M'$ be the price profile resulting from the previous steps. Note that, by construction, at p there are two groups of firms: the highest priced firms $H(p)$ charging p_h and the lowest priced firms $L(p)$ charging $p_l = p_h - \epsilon$. According to Step 2 we have two cases: either (i) the lowest priced firm are not capacity-constrained or (ii) the lowest priced firm are capacity-constrained. Let us consider the two cases separately.

Case (i): Since we are in the case such that the lowest priced firms are not capacity-constrained then the residual demand of the highest priced firm(s) is zero and so its profit. Therefore, each highest priced firm has a profitable deviation to a price $p_l - \epsilon$ for an arbitrarily $\epsilon > 0$. Such a deviation is profitable by the fact that $\pi_i(p_h) = 0 < \pi_i(p_i, p_{-i})$ for any $p_i \in (\underline{p}, p_l)$, which is the case.

By letting each highest priced firm $h \in H(p)$ to deviate we induce a new price profile $p' \in f^q(p)$ for some $q > 0$. Such a price profile is also characterized by two groups of firms: the highest priced firms $H(p')$ charging p'_h and the lowest priced firms $L(p')$ charging p'_l. If the lowest priced firms are capacity-constrained, then move to Case (ii), otherwise case (i) applies again.

Case (ii): Since we are in the case such that the lowest priced firms are capacity-constrained then the residual demand of the highest priced firm(s) is positive and so its profit.

At p_h, an highest priced firm h has a profitable deviation undercutting $p_l = p_h - \epsilon$ for some $\epsilon > 0$ when

$$\begin{aligned}
\pi_h(p_h - 2\epsilon) > \pi_h(p_h) &\iff \\
k_h(p_h - 2\epsilon) > \frac{k_h}{\sum_{j\in\Omega(p_h)} k_j}\left[D(p_h) - \sum_{j\in\Delta(p_h)} k_j\right] p_n &\iff \\
k_h p_h - \frac{k_h}{\sum_{j\in\Omega(p_h)} k_j}\left[D(p_h) - \sum_{j\in\Delta(p_h)} k_j\right] p_n > 2k_h\epsilon &\iff \\
p_h\left[1 - \frac{D(p_h) - \sum_{j\in\Delta(p_h)} k_j}{\sum_{j\in\Omega(p_h)} k_j}\right]\frac{1}{2} > \epsilon &\iff
\end{aligned}$$

Let denote by $\mathbb{A}$ the LHS of the above inequality, i.e.

$$\mathbb{A} \equiv p_h\left[1 - \frac{D(p_h) - \sum_{j\in\Delta(p_h)} k_j}{\sum_{j\in\Omega(p_h)} k_j}\right]\frac{1}{2}.$$

Note that $\mathbb{A} > 0$ when

$$1 - \frac{D(p_h) > \sum_{j \in \Delta(p_h)} k_j}{\sum_{j \in \Omega(p_h)} k_j} \iff \sum_{j \in \Omega(p_h)} k_j > D(p_h) - \sum_{j \in \Delta(p_h)} k_j \iff$$

$$\sum_{j \in \Omega(p_h)} k_j + \sum_{j \in \Delta(p_h)} k_j = K > D(p_h),$$

which holds since $p_h > \underline{p}$ by Step 1.

Since the choice of ϵ in Step 2 and Step 3 was arbitrary, we conveniently fix $\epsilon \in (0, \mathbb{A})$ such that each highest priced firm in $H(p)$ has a profitable deviation to $p_h - 2\epsilon < p_h - \epsilon$. Note we can let each highest priced firm deviates by the same ϵ. Indeed, every round a highest priced firm deviates and then $\mathbb{A}$ increases implying that ϵ is well defined.

The transition of each highest priced firm induces a new price profile $p' \in f^{\nu}(p)$ for some $\nu > 0$, characterized by two groups of firms: the highest priced firms charging p'_h and the lowest priced firms charging p'_l. Since a lowest priced firm is capacity-constrained, Case (ii) applies again.

Step 4: The iteration of previous steps constitutes a procedure which generates a sequence

$$p = p^1 \in f(p^0), p^2 \in f(p^1), ..., p^{\kappa} \in f(p^{\kappa - 1}).$$

By construction, there exists a $\kappa > 0$ such that $||p^{\kappa} - \underline{p}|| < \epsilon$, for all $\epsilon > 0$. Then, by definition of f^{∞}, it holds that $\underline{p} \in f^{\infty}(p)$. By Lemma 5, it follows that $\underline{p} \in M'$, a contradiction.

Uniqueness: Finally, we show that M is the unique MSS. By contrast, let us assume that there is another MSS M'. First, we show that $M \cap M' \neq \varnothing$. Towards a contradiction, suppose that $M \cap M' = \varnothing$. Then, by asymptotic external stability of M', for all $p \in M$ there is $p' \in M'$ such $p' \in f^{\infty}(p)$. Then, by closedness of M the intersection between the open ball around p' with radius ϵ and M is empty, i.e. $B_{\epsilon}(p') \cap M = \varnothing$. By definition of f^{∞}, there is $\kappa \in \mathbb{N}$ and a $p'' \in P$ such that $p'' \in f^{\kappa}(p)$ and $p'' \in B_{\epsilon}(p')$. By κ-fold application of *deterrence of external deviations*, it holds that $p'' \in M$, but $p'' \in B_{\epsilon}(p')$, a contradiction. Thus $M \cap M' \neq \varnothing$. In what follows we prove that $M \subseteq M'$. Equality follows from the minimality of M'. As before, we have that either the market-clearing price belong to the set M' or not.

(1): $\underline{p} \in M'$. Then by Case 1 of the minimality proof, M' contains also $M \backslash \{\underline{p}\}$.

Hence, $M \subseteq M'$.

(2): $\underline{p} \notin M'$. This possibility is ruled out by Case 2 of the minimality proof. ■

Proof of Proposition 3. By the definition of the hyper-competitive price and the market-clearing price, we have the following expressions:

$$\widetilde{p}_i = \frac{\overline{p}_1\left(D(\overline{p}_1) - K_{-i}\right)}{k_i} \text{ and } \underline{p} = \frac{\overline{p}_1\left(D(\overline{p}_1) - K_{-1}\right)}{k_1} \Longrightarrow \underline{p}\cdot k_1 + \overline{p}_1 K_{-1} = \overline{p}_1 D(\overline{p}_1).$$

Combining gives:

$$\widetilde{p}_i = \frac{\overline{p}_1 D(\overline{p}_1) - \overline{p}_1 K_{-i}}{k_i} = \frac{\underline{p}\cdot k_1 + \overline{p}_1\left(K_{-1} - K_{-i}\right)}{k_i},$$

which implies

$$\widetilde{p}_i(k_1, k_i) = \frac{1}{k_i}\cdot \underline{p}k_1 - \frac{1}{k_i}\overline{p}_1\left(k_1 - k_i\right), \tag{7}$$

for all firms $i \in N\backslash\{1\}$ with $k_i < k_1$. Total differentiation with respect to k_1 gives:

$$\frac{d\widetilde{p}_i}{dk_1} = \frac{1}{k_i}\cdot\left[\frac{d\underline{p}}{dk_1}k_1 - \frac{d\overline{p}_1}{dk_1}\left(k_1 - k_i\right) - \left(\overline{p}_1 - \underline{p}\right)\right], \tag{8}$$

which is negative because $d\underline{p}/dk_1 < 0$ and $d\overline{p}_1/dk_1 > 0$. That is, since total capacity is increasing with a rise in k_1, the market-clearing price is decreasing. Moreover, given that the market-clearing price $\underline{p}$ is decreasing, the iso-profit price of firm 1 is increasing because in this case:

$$\overline{p}_1 = \min\left\{p_1 \in P_1 | \pi_1^h(p_1) = \underline{p}\cdot k_1 \text{ with } p_1 \neq \underline{p}\right\}.$$

It therefore holds that:

$$\frac{d\widetilde{p}_i}{dk_1} = \frac{1}{k_i}\cdot[\underbrace{\frac{d\underline{p}}{dk_1}k_1}_{(-)} \underbrace{-\frac{d\overline{p}_1}{dk_1}\left(k_1 - k_i\right)}_{(-)} \underbrace{-\left(\overline{p}_1 - \underline{p}\right)}_{(-)}] < 0. \tag{9}$$

Thus, an increase in k_1 reduces the hyper-competitive prices of all firms other than i. It also leads to a reduction of the hyper-competitive price of the largest firms, *i.e.*, the market-clearing price $\underline{p}$ reduces too. In turn, this implies an increase of $\overline{p}_1$ and therefore an expansion of the MSS, all else unchanged. ■

Proof of Theorem 3. To prove that $\mathcal{K} \subset M$ when the set of pure-strategy Nash equilibria is empty, we show that: (i) $\overline{p}_1 > p_i^*$, and (ii) $\widehat{p}_i > \widetilde{p}_i$, for all $i \in N$.

Case (i): To begin, let us establish that $\overline{p}_1 > p_i^*$. We can distinguish two cases: (1) $\underline{p} > 0$, and (2) $\underline{p} = 0$.

(1) Suppose that $\underline{p} > 0$. If all price at $\underline{p}$, then all produce at capacity. Hence, there is no incentive to cut price. Since the set of pure-strategy Nash equilibria is empty, it then must hold that at least one firm is willing to hike its price. It can be easily verified that when the largest firm does not want to raise its price, none of the firms has an incentive to raise price. Hence, firm 1 has an incentive to hike its price, which implies $\underline{p} < p_1^*$. Moreover, by strict concavity of π_1^h, firm 1's residual profit function is increasing up to the (unique) profit-maximizing price and decreasing at prices in excess of p_1^* until its contingent demand is zero. Since firm 1's profits are positive at $\underline{p} > 0$, this implies that $\overline{p}_1$ is on the decreasing part of the residual profit function and therefore that $\overline{p}_1 > p_1^*$. Finally, by strict concavity of π_i^h, it holds that $p_1^* \geqslant p_i^*$, for all $i \in N \backslash \{1\}$, so that $\overline{p}_1 > p_i^*$.

(2) Now suppose that $\underline{p} = 0$. Since the set of pure-strategy Nash equilibria is empty it holds that $K_{-1} < D(0)$. Hence, firm 1 faces a strictly concave residual profit function with a unique maximizer p_1^*. Because it receives zero profits at $\underline{p} = 0$, it follows that $\overline{p}_1 = D^{-1}(K_{-1}) > p_1^*$. Moreover, following the same logic as under (1) above, $p_1^* \geqslant p_i^*$ and therefore $\overline{p}_1 > p_i^*$, for all $i \in N \backslash \{1\}$.

Case (ii): Let us now turn to the lower bound and show that $\widehat{p}_i > \tilde{p}_i$, for all $i \in N$. We again distinguish two cases: (1) $\underline{p} > 0$, and (2) $\underline{p} = 0$.

(1) Suppose that $\underline{p} > 0$. Since each firm can sell its entire capacity at $\underline{p}$, all prices below $\underline{p}$ are strictly dominated. As $\tilde{p}_i < \underline{p}$ for all firms strictly smaller than firm 1, none of these firms puts mass on its hyper-competitive price (or any lower price). Regarding the largest firm(s), recall that they have an incentive to raise their price above $\underline{p}$ in this case, which means $\pi_i^h(p_i^*) > \pi_i^h(\underline{p})$. Consequently, none of the largest sellers puts mass on prices weakly below $\underline{p}$, which is their hyper-competitive price.

(2) Now suppose that $\underline{p} = 0$. Since $K_{-1} < D(0)$, firm 1 can guarantee itself a strictly positive profit independent of the prices set by its competitors. Hence, firm 1 puts zero mass on $\underline{p} = 0$ in equilibrium. Yet, given that firm 1 prices strictly above 0, the same logic applies to firm 2. That is, this firm can guarantee itself a strictly positive profit by pricing below but arbitrarily close to the lower bound of firm 1's mixed-strategy support. Consequently, firm 2 also puts zero mass on $\underline{p} = 0$ in equilibrium. This iterative domination argument can be repeated till firm n. We conclude that when $\underline{p} = 0$, none of the firms puts mass on 0 in a mixed-strategy Nash equilibrium.

Taken together, $\widehat{p}_i > \tilde{p}_i$, for all $i \in N$, and therefore $\mathcal{K} \subset M$. ■