Sapienza Università di Roma

ISSN 2385-2755
DiSSE Working papers
[online]

WORKING PAPERS SERIES
DIPARTIMENTO DI
SCIENZE SOCIALI ED ECONOMICHE

Labor-Augmenting Technical Change and the Labor Share: New Microeconomic Foundations

Daniele Tavani, Luca Zamparelli

N. 02/2020

SAPIENZA - UNIVERSITY OF ROME
P.le Aldo Moro n.5 – 00185 Roma T(+39) 0649910563
CF80209930587 – P.IVA 02133771002

Labor-Augmenting Technical Change and the Labor Share: New Microeconomic Foundations

Daniele Tavani,[*] Luca Zamparelli[†]

Abstract

An important question in alternative economic theories has to do with the relationship between the functional income distribution and the growth rate of labor productivity. According to both the induced innovation hypothesis and Marx-biased technical change, labor productivity growth should be an increasing function of the labor share. In this paper, we first discuss the shortcomings of both theories and then provide a novel microeconomic foundation for a direct relationship between the labor share and labor productivity growth. The result arises because of profit-seeking behavior by capitalist firms that face a trade-off between investing in new capital stock and innovating to save on labor costs. Embedding this finding in the Goodwin (1967) growth cycle model, we show that: i) the resulting steady state is locally stable, and ii) unlike in the original Goodwin model, the long-run employment rate is sensitive to investment decisions. Finally, iii) we numerically show that growth cycles vanish for high elasticities of the innovation function to R&D spending.

Keywords: Endogenous Technical Change, Income Shares, Employment.
JEL Codes: E32, O33.

[*]Department of Economics, Colorado State University. 1771 Campus Delivery, Fort Collins, CO 80523-1771. Email: daniele.tavani@colostate.edu. Support from the Visiting Professor Program at Sapienza University of Rome during the Summer of 2019 is gratefully acknowledged.

[†]Department of Social Sciences and Economics, Sapienza University of Rome. Piazzale Aldo Moro, 5, Rome Italy 00185. Email: luca.zamparelli@uniroma1.it

1 Introduction

Over the last two decades, alternative growth theories have shown a pronounced interest in endogenous technical change. Among the competing explanations of the evolution of technology, several contributions have emphasized the dependence of labor productivity growth on the functional distribution of income (see among others Foley, 2003; Lima, 2004; Julius, 2006; Hein and Tarassow, 2010; Rada, 2012; Storm and Naastepad, 2012; Dutt, 2013; Dàvila-Fernandez, 2018). The general underlying rationale is based on the incentives for firms to introduce labor-saving innovations when facing high unit labor costs, which coincide with the labor share of income at the aggregate level. From an empirical standpoint, the idea that high wages foster labor-augmenting innovation has appeal in both mainstream and alternative economic circles. Among the former, the seminal work on the British industrial revolution by Robert Allen (Allen, 2009) is built around the claim that high pre-industrial wages—in combination with low energy costs—were the driving force behind the wave of mechanization that characterized the industrial takeoff in Great Britain. In the alternative literature, a comprehensive empirical analysis appears in a recent paper by De Souza (2017), who used a panel error-correction model to identify the long-run nexus between real wages and subsequent labor-augmenting innovations in manufacturing. Allen's work relies on the basic neoclassical idea of factor-substitution responding to changes in relative factor prices, while the premise of De Souza's work is in the bias of technical change as the main force behind the process of capital deepening.

There are basically two economic theories that have explicitly analyzed the microeconomic foundations of the relation between labor productivity growth and income distribution: (i) the induced innovation hypothesis first proposed by Kennedy (1964); and (ii) the theory of Marx-biased technical change presented in Michl (1999); Foley and Michl (1999, Ch. 7); Michl (2002). Most contributions involving a direct relation between productivity growth and the wage share are based – implicitly or explicitly – on either one of these two theories.

In developing the theory of induced innovation, Kennedy (1964) formally proved a conjecture by Hicks (1932): profit-seeking firms have incentives to augment the productivity of the factors becoming "more expensive" in production. The microeconomic argument goes as follows. Capitalist firms choose a profile of technical change—that is a combination of capital- and labor-augmenting innovations—so as to maximize the rate of unit cost reduction, or equivalently the rate of growth of the profit rate, subject to a technological constraint that Kennedy called *innovation possibility frontier* (IPF hereafter). The frontier describes the trade-offs between implementing capital- as opposed to labor-augmenting technological change, and it is strictly concave in order to capture a notion of increasing complexity in adopting labor- *vs.* capital-augmenting blueprints. Funk (2002) refers to the combination of myopic firm behavior and the technological constraint given by the IPF as *hypothesis of induced innovation*.[1] The firms' choice delivers a direct relation between the labor (profit) share and the rate of labor- (capital-) augmenting technical progress. The policy appeal of such result for economists working within alternative paradigms is that it provides a channel, based on microeconomic logic, through which redistribution toward labor may foster labor productivity growth in the economy.

Marx-biased technical change (MBTC), on the other hand, is a form of technical change that is simultaneously labor-saving and capital-using; that is, it designs a pattern of technology such that labor productivity increases while capital productivity decreases. It is known as 'Marx-biased' because, once coupled with a constant wage share, it delivers a falling rate of profit. Despite the broadly trendless nature of the capital-output ratio emphasized by Kaldor (1961) as one of the stylized facts of growth, MBTC appears empirically relevant since capital productivity has declined for prolonged periods of time in several industrialized countries (see for example Dumenil and Levy, 1995 and table 2.8 in Foley et al., 2019). In this context, the microeconomic foundation of the link between labor productivity growth and the labor share is provided by the criterion for the *viability* of technical change, according to which firms adopt a new technique of production if it does not reduce the profit rate at the current wage rate (Okishio, 1961). An increase in the wage share means an increase in the proportion of labor to total costs, and a technique of production that saves on labor and employs more capital becomes more likely to raise the profit rate and be viable. It follows that a higher share of labor is associated with higher labor productivity growth.

Neither of the two theories is immune to criticism, as one can expect. The induced innovation hypothesis has been criticized along two lines. On the one hand, it only explains the *direction*—i.e. the relative bias between different types of factor-augmenting new technologies—but not the *intensity* of technical change. In fact, the position of the IPF is given exogenously and does not depend on the amount of resources spent on innovation by either private firms or the public sector (exceptions are Kamien and Schwartz, 1969; Nordhaus, 1967; Zamparelli, 2015; Tavani and Zamparelli, 2020). To put it differently,

[1] In most of the literature, the choice is indeed myopic. This can be justified, as done in Funk (2002), through the occurrence of imitation by competitors following a successful new technology adoption by an individual firm. However, there are examples of infinite-horizon applications of the theory: Kamien and Schwartz (1969) that considers a decentralized partial equilibrium setting, and Nordhaus (1967) that studies the choice of both the direction and intensity of technical change by a social planner in a two-sector growth model similar to Uzawa (1961) augmented by Kennedy's IPF.

in Kennedy's world, firms have *free* access to a certain rate of technological improvement and only choose whether to distribute such new technologies between capital- and labor-augmenting innovations. One important implication is that the long-run growth rate of the economy along a balanced growth path with constant capital productivity is ultimately exogenous.

On the other hand, the very terms of the trade-off given by Kennedy's IPF are also exogenous and invariant over time. In other words, the shape of the IPF is not allowed to change, regardless of the path of innovations selected by the economy. This is true even when one introduces the optimal choice of the intensity of technical change as done, among others, in Nordhaus (1967). The combination of the two criticisms exposes the limited ability of the theory to provide: a) an explanation for the long-run growth rate of an economy and b) an account for the determination of income distribution in the long run, given that the latter ultimately depends on an exogenously given, time-invariant trade-off between relative factor-augmenting innovations. These two main criticisms were at the heart of a scathing paper by Nordhaus (1973), which marked the decline in the mainstream interest in the theory of induced bias in technology.

The theory behind MBTC suffers of similar problems. Positive labor productivity growth coupled with negative growth of capital productivity can be rationalized in two ways. Either they are taken as exogenous, so that neither the intensity nor the direction of technical change are explained; or they can be the outcome of induced innovation when the wage share is particularly high (see Foley et al., 2019, Ch. 8), in which case all the problems affecting the IPF do apply as well. Additionally, MBTC is incompatible with balanced growth as the capital-output ratio never settles to a constant value, and thus it appears ill-suited to provide a foundation for a long-run theory of growth and distribution.

In this paper, we enter the debate by providing a novel way to look at the relationship between labor-augmenting technical change and the income share of labor. Our contribution is twofold: first, the analysis overcomes some of the limitations of induced innovation and MBTC while retaining the direct relationship between the labor share and labor productivity growth, all of it grounded in microeconomic logic; second, it explores the implications of such new foundation once it is embedded in the Classical model of the growth cycle in order to assess the role of endogenous innovation in the distributive conflict that lies at the heart of Classical-Marxian economics.

We start by considering the firm-level trade-off between investing resources in capital accumulation *vs.* R&D given the size of the firm's profits. In so doing, we combine well-established insights from the endogenous growth literature—that has highlighted the role of R&D spending in fostering an economy's growth rate (see for example Aghion and Howitt, 2010)—with the Classical notion of class-based, profit-driven accumulation. Investing in both capital accumulation and labor-augmenting innovation increases future profits, although for different reasons. Capital accumulation increases net revenues given that it results in an increase in the firm's size; while labor-augmenting innovation reduces unit labor costs for given wages. An increase in the wage share has two effects: it reduces funds available for both kinds of investment and it provides an incentive to change the composition of investment in favor of R&D in order to save on more expensive labor requirements. The latter effect dominates; thus, the model generates a result similar to the induced innovation conclusion that investment in R&D—and therefore the economy's labor productivity growth rate—responds directly to the labor share.

Importantly, there are two differences between our result and the induced innovation literature. First, the trade-off faced by firms is between labor productivity growth and capital accumulation rather than between the growth rate of labor and capital productivity: we assume the latter to remain constant in the analysis, so that our contribution deals with the determination of the intensity, and not the direction, of technical change. Second, we explicitly model the tradeoff as costly, as opposed to freely available to firms. The costly—thus endogenous—nature of labor productivity growth and the constant capital-output ratio also distinguishes our result from MBTC: the economy described by our model is in balanced growth in the long run.

We can then evaluate the implications of our result for the Classical growth cycle in labor-constrained economy, which is one area of research where induced bias has been fruitfully incorporated. It is well-known that an endogenous labor-augmenting direction of technical change acts in dampening the perpetual conflict over income distribution at the heart of the Goodwin (1967) model (Shah and Desai, 1981; van der Ploeg, 1987; Foley, 2003; Julius, 2006). The reason is that directing technical change toward labor provides capitalist firms with the possibility to respond to wage increases on behalf of workers with counterbalancing labor-augmenting innovations that keep unit labor costs in check—a channel that was precluded in the original Goodwin contribution because of its very assumption of exogenous technical change. In the present context, the direct response of labor-augmenting technical progress to the share of labor will be enough to produce local stability around the balanced growth path of the economy. However, it is not clear whether convergence to the long-run position will occur monotonically or cyclically: in other words, the local stability of the steady state may or may not be associated with distributive cycles at all, not even along the transitional dynamics. The final portion of this contribution is dedicated to a numerical evaluation of the conditions on the model's main parameters—i.e. the elasticity of the innovation function and the amount of total resources available for investment in new capital stock or R&D—such that convergence to the steady-state is cyclical as opposed to monotonic. The analysis illustrates that, while for most parameters configurations the standard result of cyclical convergence occurs, when the elasticity of the innovation function is sufficiently high the transition towards the steady state becomes monotonic. The ability of firms to respond to wage increases becomes so strong that oscillations in income distribution disappear altogether, rather than simply being dampened.

Finally, our model has some interesting implications regarding its comparative statics. The long-run employment rate of the economy, which is tied up to the growth rate, is also increasing in the labor share. This marks a fundamental difference with the Goodwin (1967) model, where the steady state value of the employment rate was independent of income distribution. An increase in the saving rate will make more funds available for both accumulation and innovation, and will produce an increase in the long-run labor share. This result also holds in the original Goodwin model, where however is inconsequential for long-run employment. Here, instead, higher saving rates not only increase the workers' share of national income, but also result in a higher long-run employment rate and productivity growth in the economy. A similar effect was found in Tavani and Zamparelli (2015); but while they established it only for a calibrated model using US data, we are able to derive this result analytically given that our framework is much simpler. The other main parameter of the model has to do with the degree of labor market conflict, that is the slope of the real-wage Phillips curve. Here, and similar to the original Goodwin model, higher conflict in the labor market is inconsequential for income distribution in the long run, and only results in negative steady state employment effects.

Summing up, our contribution provides new microeconomic foundations for the direct relationship between the intensity of technical change—that is, the growth rate of labor productivity—and the share of labor. As such, it highlights a channel through which labor-friendly policies may have positive long-run growth effects; but it does not suffer of the pitfalls of induced innovation or MBTC. It does so in a parsimonious two-dimensional model of a labor-constrained economy; it shows analytically that the steady state is locally stable; and it numerically identifies conditions under which convergence to the long-run growth path is cyclical or monotonic, so that the classical growth cycle may or may not vanish.

2 Related Literature

Our paper is related to a recent and growing literature that has introduced endogenous, costly technical change in non-neoclassical models of growth. Tavani and Zamparelli

(2015) and Zamparelli (2015) study the problem of the allocation of saved profits between investment in physical capital and R&D investment in the Classical model with exogenous labor supply. Tavani and Zamparelli (2015) find investment in both capital stock and R&D as the solution of intertemporal optimization by forward-looking capitalist households. Their results differ from this contribution in two respects. First, they do not establish a direct relationship between R&D spending and the labor share. Second, they are not able to evaluate the transitional dynamics analytically; in their numerical implementation of the model calibrated to US data, the convergence to the steady state is always cyclical.

Next, and similarly to our paper, Zamparelli (2015) solves the firms' problem of capital accumulation and R&D investment through a short-run myopic profit maximization problem. However, he retains the original IPF, which mediates the relation between productivity growth and the wage share. Foley et al. (2019, Ch, 9) find the profit rate-maximizing allocation of capital between production and investment in R&D in a Classical growth model. They do obtain a direct relation between productivity growth and the labor share, but with some relevant differences relative to our framework. In fact, they posit that R&D investment can be financed by drawing down the stock of capital rather than by investing the flow of retained profits. Importantly, under their assumption an increase in the wage share does not reduce the amount of resources available to finance R&D investment. Since the potentially negative effect of a higher wage share on R&D investment is ruled out by assumption, the direct relation between the wage share and labor productivity growth in Foley et al. (2019) is much more likely to occur than in our analysis. Additionally, such a relation is embedded at the aggregate level in a conventional wage share rather than labor-constrained classical growth model.

Finally, Caminati and Sordi (2019) introduce costly endogenous technical change into a demand-led growth model where capacity utilization is at its normal level in the long-run, which places their contribution within the so-called literature on the 'supermultiplier' (see for example Serrano, 1995; Allain, 2015; Freitas and Serrano , 2015). They do find that growth and labor productivity growth are wage-led in the long-run; but differently from our contribution the short-run size of R&D investment does not depend on the wage share.

3 Basic Features of the Model

3.1 Production and Innovation

The final good Y is produced using labor L and homogeneous capital K in fixed proportions. Time is continuous, and the labor force is constant and normalized to one for simplicity.

Letting A denote (the endogenously time-varying) stock of labor-augmenting technology, and denoting the constant output/capital ratio by B, the production technique is

$$Y = \min\{AL, BK\}. \tag{1}$$

In line with the mainstream endogenous growth literature (surveyed extensively in Aghion and Howitt, 2010), we assume that the flow of labor productivity improvements $\dot{A}$ depends positively on R&D inputs and on the existing level of technology itself. Accordingly, we impose

$$\dot{A} = (R/Y)^{\alpha} A, \tag{2}$$

where R is the amount of physical output invested in R&D, and $\alpha \in (0,1)$ is the constant elasticity of innovation to R&D investment share. The linear spillover from the stock of labor-augmenting 'knowledge' to the production of new ideas is a standard assumption useful to generate endogenous growth. The normalization of R&D investment, on the other hand, is necessary to avoid explosive growth when R&D inputs consist of an accumulating factor (physical output) rather than a non-reproducible one (scientists). It is typically justified with the argument of increasing complexity of discovering new ideas, or the dilution argument of R&D investment over an increasing number of sectors (Howitt , 1999).

3.2 Income Distribution, Capital Accumulation and Technical Change

Profit maximization by firms requires to set labor and capital equal in effective units: $AL = BK$. Assume that each of the $L = BK/A$ employed workers in the economy receives the same real wage w. Denoting the share of labor in output by $\omega \equiv wL/Y = w/A$, equal to the unit labor cost, total profits are $\Pi = Y - wL = Y(1-\omega)$. The next step is the description of how resources are allocated to physical capital and R&D investment. From the standpoint of a profit-maximizing firm, the two types of investment pose a trade-off. They both increase total profits: capital accumulation increases the size of a firm's business, while innovation reduces unit labor costs in production. For this reason, the profit-maximizing composition of investment will depend on the wage share.

Next, following most of the alternative growth literature, we assume that there are two classes in society. Workers supply labor services inelastically, consume their whole income, and do not own capital stock. Capitalists own capital stock, earn profit income, consume and save. Let their constant propensity to save be denoted by $s \in (0,1)$. Saved profit incomes finance both innovation and accumulation. Letting δ be the share of saved profits invested in R&D, the growth rate of labor productivity growth is:

$$g_A \equiv \dot{A}/A = [s\delta(1-\omega)]^{\alpha}, \alpha \in (0,1). \tag{3}$$

Physical capital accumulation, on the other end, obeys:

$$g_K \equiv \dot{K}/K = s(1-\delta)(1-\omega). \tag{4}$$

We assume that firms act myopically and choose δ in order to maximize the instantaneous rate of growth of profits. This is in fact the same objective function assumed by the original induced innovation literature (Kennedy, 1964).[2] The difference lays in the choice variable: in our framework, firms choose the composition of investment between physical capital and R&D. In Kennedy's model, firms choose only the direction of technical change given the accumulation rate and the position (and shape) of the IPF. Differentiating total profits Π with respect to time we find $\dot{\Pi} = B[(1-\omega)\dot{K} + \omega K g_A]$, and the rate of growth of profits is

$$g_\Pi \equiv \dot{\Pi}/\Pi = [g_K + g_A\omega/(1-\omega)]. \tag{5}$$

Substituting from equations (3) and (4), the firms' problem is to choose δ so as to maximize $g_\Pi = s(1-\delta)(1-\omega) + [s\delta(1-\omega)]^{\alpha}\omega/(1-\omega)$. Given that the objective function is concave in δ, the first order condition is necessary and sufficient for a maximum. The resulting choice of δ satisfies

$$\delta^* = \frac{(\alpha\omega)^{\frac{1}{1-\alpha}}}{s(1-\omega)^{\frac{2-\alpha}{1-\alpha}}}. \tag{6}$$

The corresponding growth rate of labor productivity is

$$g_A^* = \left(\frac{\alpha\omega}{1-\omega}\right)^{\frac{\alpha}{1-\alpha}}, \tag{7}$$

which is increasing in the labor share. An increase in the share of labor implies lower total resources available for investment; but such a reduction is more than compensated by a reallocation in favor of expenditures that raise labor productivity growth. The incentive is provided by rising unit labor costs. This result is analogous to the implications of the induced innovation hypothesis, without relying on the innovation possibility frontier and

[2] Kennedy (1964) stated the firms' choice problem in terms of maximizing the rate of unit cost reduction. A simple duality argument shows that this is analogous to maximizing the growth rate of profits per unit of capital, i.e. the profit rate. See Julius (2006).

its shortcomings described above.

4 The Dynamical System

We can now study how the introduction of endogenous productivity growth affects the Goodwin growth cycle. The labor employment rate in the economy is $v = L = BK/A$ (recall that the total labor force is assumed to be constant and normalized to one). Since the output-capital ratio is also constant by assumption, we have

$$\frac{\dot{v}}{v} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} = s(1-\omega)\left(1 - \frac{(\alpha\omega)^{\frac{1}{1-\alpha}}}{s(1-\omega)^{\frac{2-\alpha}{1-\alpha}}}\right) - \left(\frac{\alpha\omega}{1-\omega}\right)^{\frac{\alpha}{1-\alpha}} =$$

$$= s(1-\omega) - \left(\frac{\alpha\omega}{s(1-\omega)}\right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha\omega}{1-\omega}\right)^{\frac{\alpha}{1-\alpha}}. \tag{8}$$

As it is standard, we assume that the growth rate of the real wage responds to the extent of labor market tightness as captured by the employment rate: $\dot{w}/w = f(v)$, with $f'(v) > 0$. Thus, the labor share dynamics obeys:

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{w}}{w} - g_A = f(v) - \left(\frac{\alpha\omega}{1-\omega}\right)^{\frac{\alpha}{1-\alpha}}. \tag{9}$$

4.1 Steady State and Comparative Statics

From $\dot{v} = 0$, the steady state value of the wage share is an implicit function such that

$$s = \frac{(\alpha\omega_{ss})^{\frac{\alpha}{1-\alpha}}}{(1-\omega_{ss})^{\frac{1}{1-\alpha}}}\left(1 + \frac{\alpha\omega_{ss}}{1-\omega_{ss}}\right) \equiv G(\omega_{ss}), \tag{10}$$

with $G'() > 0$. Once ω_{ss} is known, the steady state employment rate is given by

$$v_{ss} = f^{-1}\left[\left(\frac{\alpha\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{\alpha}{1-\alpha}}\right]. \tag{11}$$

Total differentiation of (10) yields $d\omega_{ss}/ds = G'(\omega_{ss}) > 0$: the wage share is an increasing function of the saving rate. The reason is that higher savings provide more resources for both capital accumulation and innovation, but the growth rate of capital rises more than labor productivity growth. In fact, the accumulation rate is linear in the saving rate; while productivity growth responds less than proportionally to higher savings given the elasticity of the innovation function, which is below one. The implication is that a higher wage share is necessary to restore balanced growth, because an increase in the share of labor lowers the resources available for capital accumulation more than the rate of technical change. On the other hand, equation (11) shows that a higher long-run value for the wage share and, in turn, higher labor productivity growth require the steady state employment rate to rise. An increase in the employment rate determines faster growth in real wages, which acts in stabilizing the wage share at a new, higher, steady state level. A positive shock to the saving rate thus produces a simultaneous rise in the wage share, productivity growth and employment. From this point of view, the comparative statics of the steady state resembles results found in Tavani and Zamparelli (2015) and Zamparelli (2015), but with the differences already emphasized in Section 2.

If, for tractability reasons, we assume a linear real-wage Phillips curve such as $f(v) = \beta v$, it follows that $v_{ss} = \frac{1}{\beta}\left[\frac{\alpha\omega_{ss}}{(1-\omega_{ss})}\right]^{\alpha/(1-\alpha)}$. The slope of the real wage Phillips curve, β, provides a measure of the degree of labor market conflict. Similarly to the original Goodwin model, higher conflict in the labor market only reduces the steady state employment rate; but it has no effect on income distribution and productivity growth.

4.2 Local Stability Analysis

Linearization of the dynamical system formed by equations (8) and (9) around its steady state position yields the Jacobian matrix:

$$J(v_{ss}, \omega_{ss}) = \begin{bmatrix} \dot{v}_v & \dot{v}_\omega \\ \dot{\omega}_v & \dot{\omega}_\omega \end{bmatrix}_{ss}.$$

Let us now evaluate the various entries of the Jacobian at the steady state. We have:

$$\dot{v}_v = 0,$$

$$\dot{v}_\omega = v_{ss}\left[-s - \frac{\alpha^{\frac{1}{1-\alpha}}}{1-\alpha}\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{\alpha}{1-\alpha}}\frac{1}{(1-\omega_{ss})^2} - \frac{\alpha^{\frac{1}{1-\alpha}}}{1-\alpha}\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{2\alpha-1}{1-\alpha}}\frac{1}{(1-\omega_{ss})^2}\right]$$

$$= -\frac{1}{\beta}\left[s\left(\frac{\alpha\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{\alpha}{1-\alpha}} + \frac{\alpha^{\frac{1+\alpha}{1-\alpha}}}{1-\alpha}\frac{\omega_{ss}^{\frac{2\alpha-1}{1-\alpha}}}{(1-\omega_{ss})^{\frac{3-\alpha}{1-\alpha}}}\right] < 0,$$

$$\dot{\omega}_v = \beta\omega_{ss} > 0,$$

$$\dot{\omega}_{\omega} = -\frac{\alpha^{\frac{1}{1-\alpha}}}{1-\alpha}\frac{\omega_{ss}^{\frac{\alpha}{1-\alpha}}}{(1-\omega_{ss})^{\frac{1}{1-\alpha}}} < 0.$$

The Jacobian has negative trace (TrJ) and positive determinant ($DetJ$). Hence, there are two distinct eigenvalues with real parts that add up to a negative number and are of the same sign. This can be true only if the two eigenvalues are negative, which is necessary and sufficient for local stability of the steady state.

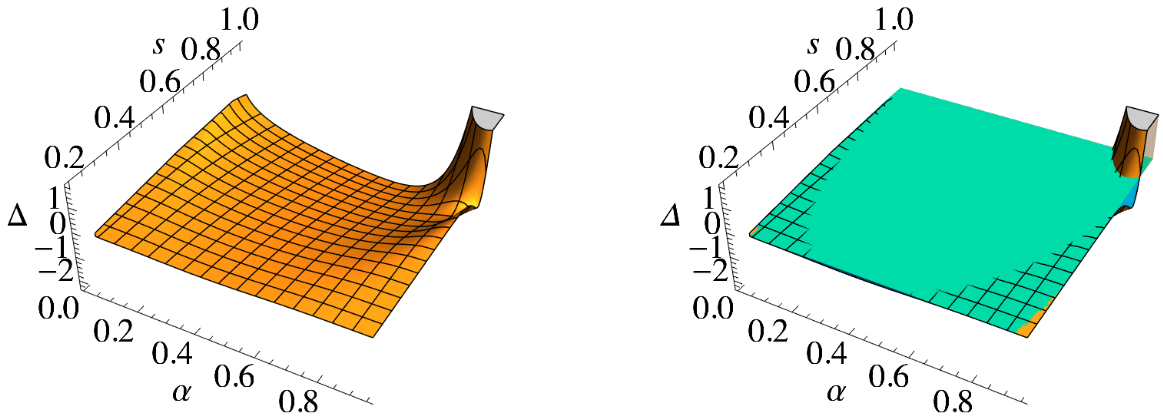
4.3 Cyclical vs. Monotonic Convergence to the Steady State

To understand whether the convergence to the steady state is monotonic or oscillatory, we need to look at the characteristic equation of the Jacobian matrix, which, given eigenvalues λ, is $\lambda^2 - TrJ\lambda + DetJ = 0$. What matters is the sign of the discriminant of the characteristic equation, that is $\Delta = (TrJ)^2 - 4DetJ$. If the discriminant is negative, the eigenvalues of the Jacobian will contain imaginary roots, and there will be oscillations in the transitional dynamics of the employment rate and the labor share before the steady state is reached. The discriminant can be calculated as

$$\Delta = \left(\frac{\alpha^{\frac{1}{1-\alpha}}}{1-\alpha}\frac{\omega_{ss}^{\frac{\alpha}{1-\alpha}}}{(1-\omega_{ss})^{\frac{1}{1-\alpha}}}\right)^2 - 4\omega_{ss}\left[s\left(\frac{\alpha\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{\alpha}{1-\alpha}} + \frac{\alpha^{\frac{1+\alpha}{1-\alpha}}}{1-\alpha}\frac{\omega_{ss}^{\frac{2\alpha-1}{1-\alpha}}}{(1-\omega_{ss})^{\frac{3-\alpha}{1-\alpha}}}\right], \qquad (12)$$

where ω_{ss}, and in turn Δ, are functions of (α, s) only. Given that the system cannot be evaluated analytically, we proceed to a numerical evaluation of the discriminant as follows: i) we let α vary in small steps from .01 to .95; ii) we let s vary in .01 steps between .2 and 1. On the one hand, we excluded values very close to 1 for the innovation elasticity α because the profit-maximizing R&D intensity δ^* is not defined as α approaches 1. On the other hand, we excluded small values for the saving rate for two reasons: first, $\delta^* \to \infty$ as $s \to 0$ as it is clear from equation (6); second, as we show in the Appendix, the labor share is not defined in the numerical implementation of the model for values of the saving rate below .2. We can then evaluate the Jacobian and the corresponding discriminant $\Delta(\alpha, s)$ at each of the steady states pinned down by any pair of the two parameters of interest. We can finally display the discriminant as a function of (α, s) in a three-dimensional plot. The left panel of Figure 4.3 displays the three dimensional plot of the discriminant, while the right panel also plots the hyperplane going through $\Delta = 0$ in green. As it can be seen, the discriminant is negative almost everywhere, save for a small region corresponding to very high values of the innovation elasticity α and of the investment rate s. This means that for most parametric configurations the convergence to the steady state is cyclical: thus, it reproduces

Figure 1: The discriminant of the Jacobian matrix for varying (α, s). The green area in the right panel displays the hyperplane going through $\Delta = 0$.



the standard dynamics of the growth cycle with induced technical change. However, when R&D returns become very high, with α roughly above 0.8, the dynamic transition towards the steady state becomes monotonic and cycles disappear. In the distributive conflict responsible for the emergence of cycles, capitalists are now so effective in responding to wage increases that oscillations in income distribution are not simply dampened but vanish altogether.

5 Concluding Remarks

Our analysis has provided a novel way to look at the interaction between labor-augmenting technical change and income shares—and its implications for distributive conflict in a Classical growth model—that explicitly considers the microeconomics of investment in both accumulation and innovation at the firm level. Our main result is that, provided that firms face trade-offs in investing their profit earnings in accumulation of new capital stock *vis à vis* innovating to save on labor costs, an increase in the labor share raises R&D spending in labor-augmenting innovation and therefore the growth rate of labor productivity. We find this result important for two reasons: i) it pertains to the intensity, and not to the direction, of technical change, and ii) it does not suffer of the shortcomings of either the induced innovation hypothesis or the notion of MBTC.

We then embedded this result in the Classical growth cycle. We proved analytically that the steady state is locally stable, in line with the literature on induced bias in technical change and distributive conflict (Shah and Desai, 1981; van der Ploeg, 1987; Foley, 2003;

Julius, 2006). We showed that increases in the saving rate not only improve the workers' share of national income—as it was already the case in the original Goodwin (1967) model—but also result in a higher employment rate in the long run. Since technical change is exogenous in the Goodwin model, there is only one level of real wage growth and, in turn, of the employment rate that can stabilize the wage share. In our framework, on the contrary, labor productivity growth depends on R&D investment, so that both technical change and the employment rate are endogenous, and both will increase with higher savings. Finally, we have shown numerically that convergence to the steady state can become monotonic, provided that the elasticity of innovation to R&D investment is sufficiently high. In fact, once endowed with an extremely productive innovation technology, firms become so powerful in responding to wage increases that the distributive cycles vanish completely.

More work needs to be done toward increasing the policy relevance of this framework. In particular, our simple model points to the need for identifying specific policy levers—beyond the basic investment channel discussed here—that may result in a higher wage share in steady state, and under which conditions such levers will result in faster growth without hurting long-run employment. The recent empirical literature on minimum wage reforms and employment (see for example Dube et al, 2016; Gengiz et al., 2019) has shown that increases in the minimum wages seem to produce little if any effects on long-run employment, but have substantial positive welfare effects for low- and middle-income earners. Extending the Classical model of growth and distribution with endogenous technical change to incorporate a more explicit role for labor market policies appears to be a promising area for future research.

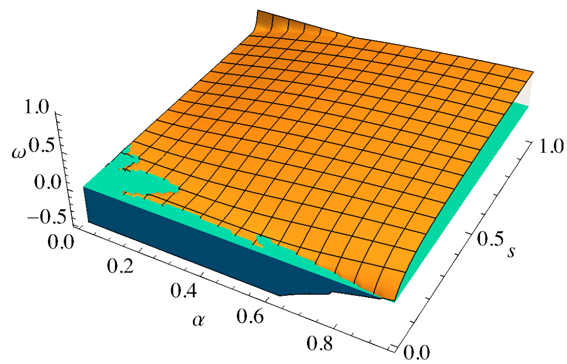
A Steady State Labor Share and the Saving Rate

As already mentioned in the text, equation (10) identifies an implicit function $\omega_{ss}(\alpha, s)$ that can be evaluated numerically letting the underlying parameters vary in small intervals. The plot below shows, similarly to the right panel in Figure (4.3), that the steady state labor share takes values below zero when the saving rate is below .2, which justifies limiting the numerical evaluation of the discriminant as done in the text.

References

Aghion, P., and Howitt, P., 2010: *The Economics of Growth* Cambridge, MA: MIT Press. 1, 3.1

Figure 2: The steady state labor share for varying (α, s).



Allain, O. (2015): Tackling the instability of growth: A Kaleckian-Harrodian model with an autonomous expenditure component. *Cambridge Journal of Economics* 39 (3): 1351-1371. 2

Allen, R. E. (2009): *The British Industrial Revolution in Global Perspective*. Cambridge, UK: Cambridge University Press. 1

Caminati, M. and S. Sordi (2019): Demand-led growth with endogenous innovation. *Metroeconomica* 70 (3): 405-422. 2

Dávila-Fernández, M.J. (2018): Alternative Approaches to Technological Change when Growth is BoPC. Department of Economics University of Siena 795, University of Siena. 1

De Souza, J. P. A. (2017) Real Wages and Labor-Saving Technical Change: Evidence from a Panel of Manufacturing Industries in Mature and Surplus Economies. *International Review of Applied Economics* 31 (2): 151-172. 1

Drandakis, E.M., Phelps, E. (1965): A model of induced invention, growth and distribution. *Economic Journal* 76 (304): 823–840.

Dube, A., William Lester, T., Reich, M. (2016): Minimum Wage Shocks, Employment Flows, and Labor Market Frictions. *Journal of Labor Economics* 34 (3): 663-704. 5

Duménil G., and Lévy D. (1995): A stochastic model of technical change: an application to the US economy, 1869-1989. *Metroeconomica* 46 (2): 213–45. 1

Dutt, A.K. (2013): Endogenous technological change in classical–Marxian models of growth and distribution. In T. Michl and A. Rezai (eds.), *Social Fairness and Economics: Essays in the Spirit of Duncan Foley* (pp. 264–285). New York, NY: Routledge. 1

Foley, D. K., Michl, T. R., (1999): *Growth and Distribution*. Cambridge, MA: Harvard University Press. 1

Foley, D. K., Michl, T. R., Tavani, D., (2019): *Growth and Distribution, Second Edition.* Cambridge, MA: Harvard University Press. 1, 2

Foley D. (2003): Endogenous technical change with externalities in a classical growth model. *Journal of Economic Behavior and Organization*, 52: 167–89. 1, 5

Freitas, F., & Serrano, F. (2015). Growth rates and level effects, the stability of the adjustment of demand to capacity, and the Sraffian supermultiplier. *Review of Political Economy*, 27(3): 258-281. 2

Funk P. (2002): Induced innovation revisited. *Economic*a 69: 155-71. 1, 1

Gengiz, D., Dube, A., Lendner, A., Zipperer B., 2019. The effect of minimum wages on low-wage jobs. *Quarterly Journal of Economics* 134 (3): 1405-1454. 5

Goodwin R (1967) A growth cycle. In: Feinstein C. (ed.) *Socialism, capitalism, and economic growth*. Cambridge, UK: Cambridge University Press. (document), 1, 5

Hein, E., and Tarassow, A. (2010): Distribution, aggregate demand and productivity growth: theory and empirical results for six OECD countries based on a post-Kaleckian model. *Cambridge Journal of Economics* 34 (4): 727-754. 1

Hicks, J (1932): *The Theory of Wages*. London, UK: McMillan. 1

Howitt, P. (1999): Steady Endogenous Growth with Population and R. & D. Inputs Growing. *Journal of Political Economy* 107 (4): 715-730 3.1

Julius, A. J., (2006): Steady State Growth and Distribution with an Endogenous Direction of Technical Change. *Metroeconomica* 56 (1): 101-125. 1, 2, 5

Kaldor, N. (1961): Capital accumulation and economic growth. In F.A. Lutz and D.C. Hague, Eds., *The Theory of Capital Accumulation*, London. 1

Kamien M., Schwartz N. (1969): Induced factor augmenting technological change from a microeconomic viewpoint. *Econometrica* 37(4): 668–684. 1, 1

Kennedy C. (1964): Induced bias in innovation and the theory of distribution. *Economic Journal* 74 (295): 541–47. 1, 3.2, 2

Lima, G. T. (2004): Endogenous technological innovation, capital accumulation and distributional dynamics. *Metroeconomica* 55 (4): 386–408. 1

Michl, T. (1999) Biased technical change and the aggregate production function. *International Review of Applied Economics* 13(2): 193-206. 1

Michl, T. (2002) The fossil production function in a vintage model. *Australian Economic Papers* 41(1): 53-68. 1

Nordhaus, W. D. (1967): The Optimal Rate and Direction of Technical Change, in Shell, K., ed. *Essays on the Theory of Optimal Economic Growth*. Cambridge, MA: MIT Press. 1, 1

Nordhaus, W. D., (1973): Some Skeptical Thoughts on the Theory of induced innovation. *Quarterly Journal of Economics* 87 (2): 208-219. 1

Okishio, N., 1961. Technical Change and the Rate of Profit. *Kobe University Economic Review* 7: 85-99. 1

Rada, C. (2012) Social security tax and endogenous technical change in an economy with an aging population. *Metroeconomica* 63 (4): 727-756. 1

Serrano, F. (1995). Long-period effective demand, and the Sraffian supermultiplier. *Contributions to Political Economy* 14: 67– 90 2

Shah, A., Desai, M., 1981. Growth cycles with induced technical change. *Economic Journal* 91: 1006–1010. 1, 5

Storm, S. and C. W. M. Naastepad (2012). *Macroeconomics Beyond the NAIRU*, Cambridge, MA: Harvard University Press. 1

Tavani, D., and Zamparelli, L. (2015) Endogenous technical change, employment and distribution in the Goodwin model of the growth cycle. *Studies in Nonlinear Dynamics and Econometrics* 19 (2): 209-226. 1, 2, 4.1

Tavani, D., and Zamparelli, L. (2017) Endogenous Technical Change in Alternative Theories of Growth and Distribution. *Journal of Economic Surveys* 31 (5): 1272-1303,

Tavani, D., and Zamparelli, L., (2020) Growth, Income Distribution, and the 'Entrepreneurial State.' *Journal of Evolutionary Economics*, 30 (1): 117-141. 1

Uzawa, H. , 1961. On a Two-Sector Model of Economic Growth. *Review of Economic Studies* 29 (1): 40-47. 1

van der Ploeg, F., (1987): Growth Cycles, Induced Technical Change, and Perpetual Conflict over the Distribution of Income. *Journal of Macroeconomics*, 9: 1-12. 1, 5

Zamparelli, L., 2015: Induced Innovation, Endogenous Technical Change and Income Distribution in a Labor-Constrained Model of Classical Growth, *Metroeconomica* 66 (2): 243-262. 1, 2, 4.1