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Technical Change in Alternative Theories of Growth*

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Abstract

This paper investigates alternative ways of introducing technological progress in heterodox theories of economic growth. We model technical change as: i) exogenous and costless; ii) a positive externality of capital accumulation, the wage share or the employment rate; iii) endogenous and costly. We implement these formalizations in Classical growth theories, where investments coincide with full capacity savings, and Keynesian theories where capital accumulation is demand constrained. We also distinguish between abundant and inelastic labor market closures. We discuss the outcomes of these models in terms of long-run growth, functional income distribution and employment.

Keywords: Technical change, Heterodox growth models, R&D, Factor income shares, Employment

JEL Classification: D24, E25, D33, O30, O41

1 Introduction

Since the inception of modern political economy, technical change has played a prominent role in the analysis of economic growth and development. In the introduction to *The Wealth of Nations,* Adam Smith singled out labor productivity ('the skill, dexterity, and judgment with which .. labour is generally applied') as the ultimate source of economic opulence (Smith, 1776). Classical economists and Marx also understood that innovation and technology are not simply a fundamental driver of economic growth, but they are strongly intertwined with the dynamics of income distribution and employment. Both Ricardo and Marx recognized the role of profitability in eliciting

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innovation as new techniques of production would be implemented only if they increased the profit rate; while Ricardo in the famous chapter "On Machinery" suggested that technical change could harm employment (‘substitution of machinery for human labour, is often very injurious to the interests of the class of labourers’ Ricardo , 1821[1951-1973], Works I: 388).

This chapter investigates alternative ways of introducing technological progress in heterodox theories of economic growth. In particular, the focus will be on how the interaction of different formalizations of technical change and alternative closures of the model affects the economy’s growth, functional income distribution and employment in the long run.

To restrict the scope of our inquiry, we must first clarify what we mean by heterodox growth theory. We define as heterodox any theory not based on the neoclassical general equilibrium theory paradigm. In this way, we rule out both the exogenous growth model based on perfect competition, where factors of production are substitutable, fully employed and paid according to the their marginal products, and endogenous growth models based on imperfect competition, where factors of production are still fully employed and substitutable but where the existence of market power precludes marginal cost pricing. Within this definition, we will look at those theories that allow for long-run steady states featuring the so-called Kaldor facts (Kaldor, 1961). Specifically, we will restrict our analysis to fully adjusted positions where: factor income shares and the output-capital ratio are constant; technical change is purely labor-saving; and the capital-labor ratio increases at a constant rate. We will distinguish between Classical growth theories where investments coincide with full capacity savings and Keynesian theories where capital accumulation is demand constrained. Since the focus is on balanced growth, we will not discuss the prominent role of technical change in Evolutionary growth theory.

With respect to the formalization of technical change, we will explore three alternatives. The first one assumes that it is exogenous and costless. A second option models it as a positive externality of other economic variables; in particular, capital accumulation, the wage share and the employment rate may contribute to labor productivity growth. Finally, we will explore the possibility that technical change is endogenous and costly because it requires R&D investment, either private or public.

The rest of the paper is organized as follows. Section 2 lays out the analytical structure and the main assumptions of the models we investigate. Section 3 discusses Classical growth under the two alternative specifications of abundant and scarce labor supply. Section 4 explores three possible versions of Keynesian growth: a Kaleckian closure with exogenous income distribution and endogenous capacity utilization; a Harrod-Kaldorian variant with endogenous income distribution and normal, exogenous, utilization; and, finally, a Sraffian one with exogenous distribution and normal utilization. Section 5 discusses the possibility that costly R&D is paid by the public sector. Section 6 concludes summarizing the main results.

2 Analytical Framework

In this section, we outline the analytical structure of the economy and the assumptions on class division, investment, labor supply and technical change.

2.1 Production

Final output Y is produced according to a Leontief production function that combines capital K and labor L in fixed proportions:

$$Y = min[uBK, AL], \tag{1}$$

where B denotes the ratio of potential output (Y^p) to capital, A is labor productivity, and $u \equiv Y/Y^p$ is a measure of capacity utilization. The zero elasticity of substitution between factors of production implies a radical departure from the neoclassical growth model. First, with respect to income distribution, factors cannot be paid according to their marginal products, which are not defined at the profit maximizing factor choice $uBK = AL$. Second, it allows for equilibrium labor unemployment since there are no endogenous market mechanisms reconciling labor demand $L = uBK/A$ to labor supply N. When $L < N$ and $u = 1$, unemployment is purely technological, or structural. When $BK/A < N$ and $u < 1$, both technology and lack of aggregate demand contribute to unemployment.

2.2 Social classes and income distribution

There are two classes of households in the economy: capitalists own the means of production, receive profit income Π from the ownership of the capital stock, and have constant propensity to save s. Workers supply labor, earn a real wage w, and do not save. If we denote the wage share as $\omega \equiv wL/Y = w/A$, capitalists' profits will be

$$\Pi = Y(1 - \omega) = uBK(1 - \omega). \tag{2}$$

2.3 Investment

We will distinguish alternative growth theories according to the investment functions they assume. The first distinction is between supply- and demand-side theories. Supply-side theories assume that investments (I) coincide with savings (S) at full capacity: $I = sY^p(1 - \omega)$. We define $g_K \equiv I/K$ the actual growth rate of capital, which in this case yields:

$$g_K = sY^p(1 - \omega)/K = sB(1 - \omega). \tag{3}$$

In contrast, investment demand is independent of saving behavior in demand-side theories. We distinguish between exogenous and induced theories of investment. Exogenous theories posit that (at least some share) of investment does not depend on income. We can borrow from the post-Keynesian tradition and assume that, in addition to an autonomous component, investment is a positive function of measures of aggregate demand and profitability:

$$g_K = \gamma_0 + \gamma_1 u + \gamma_2(1 - \omega). \tag{4}$$

Kaldor (1956, p.94) clarified that the Keynesian principle of aggregate demand can be alternatively used to provide either a theory of the level of economic activity or of income distribution. We will use the investment function in (4) to discuss two alternative cases. In the first case, we will take income distribution as given to find the equilibrium level of capacity utilization. In the second one, we will assume that capacity utilization is at its normal level ($u = u_n = 1$) to solve for the equilibrium wage share. We will refer to these cases as, respectively, Kaleckian and Harrod-Kaldorian.[1]

Finally, induced theories of investment assume it is an increasing function of output:

$$g_K = hY/K = huB. \tag{5}$$

We will explore this function in the context of the so-called 'Sraffian supermultiplier'.

2.4 The two Harrod's problems

Modern growth theory conventionally begins with Roy Harrod's (Harrod, 1939) attempt to extend the Keynesian analysis to the long-run. He introduced the notions of 'warranted' growth rate (g_K^s) as the growth rate that ensures the saving-investment equality, and 'natural' growth rate (g^p) as the unemployment stabilizing rate, thus equal to the sum of population and labor productivity growth. He discussed the issues connected to the difference between actual, warranted and natural growth rate. We do not analyze the solutions that alternative theories have offered to reconcile the three growth rates (see Blecker and Settterfield (2019) for a recent account). We will simply assume that the economy is in a long-run state where both the goods market and the labor market are in equilibrium, so that the three growth rates equalize.

Given our notation and assumptions, the warranted growth rate is

$$g_K^s = S/K = suB(1-\omega), \tag{6}$$

while the natural rate of growth is

$$g^p = \dot{N}/N + \dot{A}/A, \tag{7}$$

with the additional hypothesis that the labor force is a constant fraction of population. The following two conditions must hold simultaneously in a long-run steady state equilibrium:

$$g_K = suB(1-\omega) = \dot{N}/N + \dot{A}/A. \tag{8}$$

We will index by ss the values of the endogenous variables that solve (8).

2.5 Labor supply

We will explore two alternative assumptions regarding labor supply. Under the first assumption,

[1] The Kaldorian growth model has mostly adopted an open economy framework and has emphasized the role of export as a source of aggregate demand (see Blecker 2022 for a recent survey). In that context, since labor productivity growth improves an economy's competitiveness and access to global markets it is considered a fundamental determinant of investment. Since we assume a closed economy, that channel is precluded and the investment function ultimately depends on profitability.

labor supply is inelastic as it grows at a constant exogenous rate n, so that $\dot{N}/N = n$. We will see that, depending on the formalization of technical change, exogenous labor supply may constrain capital accumulation and growth while affecting income distribution. On the other hand, labor supply can be infinitely elastic, or abundant, and capable of instantaneously accommodating labor demand, that is, it is endogenous to economic growth: $\dot{N}/N = \dot{L}/L \equiv g_L$. An endogenous, or abundant, labor force typically applies to emerging economies, where large rural or informal sectors provide ample labor reserves to be drawn into the formal economy. Still, even mature economies can be relatively unconstrained by labor supply through loose migration policies.

2.6 Technical change formalizations

Equation (8) shows that in a long-run equilibrium technical change increases labor productivity while leaving capital productivity constant. In fact, this is a well-established result in growth theory, known at least since Uzawa (1961). For this reason, we will mostly (but see Section 3.3) analyze formalizations of technical progress that simply leave the potential-output to capital ratio constant while producing positive labor productivity growth rates $\dot{A}/A \equiv g_A$.

We will distinguish three approaches to technical change. The simplest way to model technical change is by assuming it is exogenous. It is costless and occurs in the economy independently of economic activity and of the level of any economic variable: $g_A = a$. A first attempt to make technical change endogenous treats it as a positive externality of some economic variable x: $\dot{A} = H(x)A$. In the heterodox literature, we can find at least three variables that have been associated to positive effects on productivity growth. The Kaldor-Verdoorn law (Verdoorn, 1949; Kaldor, 1957), or technical progress function, states that technical change is an increasing function of either (absolute or per-capita) capital accumulation or output growth. It is based on the notions of economies of scale, macroeconomic increasing returns, and learning by doing. We restrict our attention to one linear version of the law that has productivity growth depending on the capital growth rate:

$$g_A = \varphi_0 + \varphi_1 g_K. \tag{9}$$

A second option makes labor productivity growth a positive function of the wage share. This notion is founded both in the classical-Marxian analysis of the choice of techniques and in the induced innovation literature discussed below in Section 3.3. The basic intuition relies on the incentive to introduce labor-saving innovations by firms facing rising labor costs, that is higher wage shares. We simply assume a reduced form that posits a direct positive relationship from the wage share to the growth rate of labor productivity:

$$g_A = f(\omega), \ f' > 0, \tag{10}$$

as in Taylor (1991) and Dutt (2013a). However, we will show below that this relation can be microfounded if we let firms optimally choose the direction or the intensity of technical change. We will refer to to this formalization as the Classical-Marxian motive or induced innovation theory. The theory of Marx-biased technical change delivers a similar relation between productivity growth and the wage share, but we will not explore it as it produces unbalanced growth.

A third alternative points to labor market tightness as a source of technical change (Dutt, 2006; Flaschel and Skott, 2006; Sasaki, 2010; Palley, 2012; Setterfield, 2013), since labor shortages would provide the incentive to adopt labor-saving innovations. The relevant variable in this case is the employment rate defined as $e \equiv L/N$, so that productivity growth can be represented as

$$g_A = q(e), \; q' > 0. \tag{11}$$

Note that this specification of technical change is only relevant with exogenous labor supply. When labor supply is endogenous, labor is never scarce and its limited supply cannot provide any incentive to innovate. We will treat each type of externality as a single source of technical chance, but multiple papers have combined two of them in making technical change endogenous (just as examples see Taylor et al. 2019; Palley 2019; Michl and Tavani 2022; Fazzari et al. 2020; Allain 2021)

Observe that in the three specifications of technical change as an externality (9), (10), and (11) we have assumed a positive linear spillover from the existing level of technology to the productivity of the factor responsible for technical progress. This means that, over time, given levels of capital accumulation, wage share or employment rate produce ever increasing rises in productivity growth. We are not aware of arguments in the literature that address the plausibility of this issue.

As an alternative, we can assume that technical change is the outcome of R&D investment, and is therefore endogenous and costly. Either physical output or labor inputs must be employed to raise productivity growth. We will follow the first route and assume that if R is the amount of final good invested in R&D, productivity growth follows

$$g_A = (R/K)^{\vartheta}, \tag{12}$$

with $\vartheta \in (0, 1)$ and the normalization of investment by capital is necessary to avoid explosive growth. We assume different specifications for R in supply- and demand- side models. In the former, R is a share of full capacity saving so that $R = \delta s B K(1 - \omega)$, where $\delta \in (0, 1)$ will be a choice variable for the optimizing firm. In the latter case, we will simply assume that R&D investment is a positive function of output: $R = \delta u B K$. We will not assume that firms choose δ optimally for two reasons: first, profit-maximizing microfoudations do not belong to the Keynesian tradition; second, and most importantly, the optimal choice of investment (in physical capital or R&D) is problematic without knowing future sales, as it is the case in a Keynesian economy (though see Caminati and Sordi (2019)).

Finally, we will conclude by discussing models where R&D investments are financed through taxation by the public sector.

3 Classical Growth

Most models belonging to the classical tradition assume Say's law, that is supply creates its own demand. This means that investment coincides with full capacity saving, which in our notation yields $g_K = g_K^s = sB(1 - \omega)$, where we have imposed $u = u_n = 1$. The main distinction within the classical tradition regards the labor supply assumptions. We start by discussing the case of abundant labor to turn next to exogenous labor supply growth.

3.1 Abundant labor

Classical political economy was born during the early stages of the industrial revolution. At the time, labor in the rural sector was abundant and could always accommodate the manufacturing sector's demand for labor. Growth was constrained by capital accumulation rather than the availability of labor. The abundance of labor was also thought to exert downward pressure on real wages, which would be constant at the exogenous subsistence level. In fact, real wages stagnated at least until the second half of the 19th century (Allen , 2009). In the context of labor productivity growth, we will not assume constant real wages as given, or the wage share would trend toward zero. We will rather take the wage share as fixed at a *conventional* value (Foley and Michl, 1999): $\omega = \bar{\omega}$. The corresponding long-run equilibrium is

$$g_K = sB(1 - \bar{\omega}) = g_{L,ss} + g_A. \tag{13}$$

In this framework, aggregate growth is fully determined by full capacity saving while the role of technical change consists in determining employment growth. Given capital accumulation, there exists a trade-off between accommodating output growth through employment or productivity growth. Faster technical change means faster real wage growth that benefits a smaller pool of workers. This is true both with exogenous technical change $g_{L,ss} = sB(1 - \bar{\omega}) - a$, or if we model productivity growth according to (9): $g_{L,ss} = sB(1 - \bar{\omega})(1 - \varphi_1) - \varphi_0$; or (10): $g_{L,ss} = sB(1 - \bar{\omega}) - f(\bar{\omega})$. In contrast, the Kaldor-Verdoorn law and the induced innovation theory imply opposite responses to distributional shocks. Under the latter assumption a rise in the wage share immediately makes productivity growth faster ($f_\omega > 0$), while a higher wage share slows down the pace of technical change by reducing capital accumulation if the Kaldor-Verdoorn law holds.

3.2 Exogenous labor supply

Let us now assume exogenous labor supply. With exogenous technical change, the model produces the Goodwin's (1967) growth cycle steady state:

$$sB(1 - \omega_{ss}) = n + a.$$

Income distribution necessarily becomes endogenous as it is responsible for the adjustment of the economy's saving to the exogenous natural growth rate. A rise in productivity growth reduces the economy's labor demand and exerts a downward pressure on real wages, which results in a lower wage share. The saving rate has an opposite effects on the wage share as its rise yields higher saving growth, which immediately translates into higher capital accumulation and labor demand.

Next, we formalize technical change as an externality. When g_A follows (9), balanced growth yields

$$sB(1 - \omega_{ss}) = \frac{n + \varphi_0}{(1 - \varphi_1)}, \tag{14}$$

with $g_{A,ss} = (\varphi_0 + n\varphi_1)/(1 - \varphi_1)$. This result is quite similar to the exogenous technical change outcome. The labor share is an increasing function of the saving rate, and a decreasing function of productivity growth. While technical change is not exogenous, its equilibrium value is ultimately determined by technological and demographic parameters only, despite being affected by capital accumulation and the saving rate during the transition to the steady state.

When labor productivity growth follows the induced innovation motive as in (10), the wage share is still the endogenous variable in the long-run equilibrium

$$sB(1 - \omega_{ss}) = n + f(\omega_{ss}), \tag{15}$$

and it is still an increasing function of the the saving preferences as total differentiation of (15) shows: $d\omega_{ss}/ds = B(1 - \omega_{ss})/(f'(\omega_{ss}) + sB) > 0$. A higher saving rate increases capital accumulation and labor demand; a higher wage share is then necessary to keep the employment rate stable as it simultaneously increases labor productivity growth and lowers the growth rate of saving. The interesting twist is that long-run technical change and growth become positive functions of the saving rate. A rise in the saving rate increases accumulation, which, however, is constrained by labor supply. This makes the wage share increase; but the higher wage share raises labor productivity growth and the natural growth rate through its effect on the wage share $g_{A,ss} = f[\omega_{ss}(s)]$,$g'_{A,s} > 0$. In the end, the economy stabilizes at a higher growth rate as the warranted growth rate adjusts to the higher natural growth rate.[2]

Finally, if technical progress follows (11) the long-run equilibrium becomes

$$sB(1 - \omega_{ss}) = n + h(e_{ss}), \tag{16}$$

and by itself it cannot determine both income distribution and the employment rate. We have two options. We can go back to the conventional wage share model and assume $\omega_{ss} = \bar{\omega}$. At that point, the equilibrium condition determine e_{ss} and productivity growth as negative function of the exogenous wage share. As an alternative, we can follow the original Goodwin (1967) model, where real wages growth is a positive function of the employment rate, as a tighter labor market raises workers' bargaining strength, say $g_w = m(e)$. Since wages and labor productivity must grow at the same rate to stabilize the wage share, then $m(e_{ss}) = h(e_{ss})$ fixes the equilibrium employment rate. Labor productivity growth is thus determined in the labor market independently of the saving rate, which, on the other hand, still has a positive effect on the equilibrium wage share through (16).

We now consider the possibility that technical change is costly, that is it requires resources $R = \delta sBK(1 - \omega)$ to produce labor productivity growth $g_A = (\delta sB(1 - \omega))^{\vartheta}$. The long-run equilibrium condition must be emended to take into account that the share δ of saved profits is spent in R&D investment:

$$(1 - \delta)sB(1 - \omega_{ss}) = n + (\delta sB(1 - \omega_{ss}))^{\vartheta}. \tag{17}$$

[2] We can see that the net effect of a higher saving rate and a lower profit share on the warranted growth rate is positive: $\frac{d}{ds}g^s_K = B(1 - \omega) - sBd\omega/ds = B(1 - \omega)(1 - sB/(sB + f')) > 0$, since $sB/(sB + f') < 1$.

Capitalists face a trade-off regarding the allocation of their savings. They can either increase the capital stock or labor productivity growth. If we assume like in Tavani and Zamparelli (2021) that they choose δ to maximize the rate of growth of profits $g_K + g_A\omega/(1-\omega)$, we find $\delta = \frac{1}{sB(1-\omega)}\left(\frac{\vartheta\omega}{1-\omega}\right)^{\frac{1}{1-\vartheta}}$ and $g_A = \left(\frac{\vartheta\omega}{1-\omega}\right)^{\frac{\vartheta}{1-\vartheta}}$. The maximizing choice of the size of R&D investment makes labor productivity growth become a positive function of the wage share, and in fact provides a microfoundation for (15). The intuition is straightforward, the higher the unit labor cost the higher the incentive to save labor inputs relative to increase the capital stock. If we then plug the optimal values of δ and g_A into the long-run equilibrium condition we find

$$sB(1-\omega_{ss}) = n + \left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{\vartheta}{1-\vartheta}}\left(\frac{1-\omega_{ss}(1-\vartheta)}{1-\omega_{ss}}\right), \tag{18}$$

which is just a specific case of (15) and confirms that the saving rate has a positive effect on the equilibrium wage share, which, in turn, raises productivity growth.

3.3 The induced innovation hypothesis

In the context of the Classical model with exogenous labor supply we need to discuss the induced innovation hypothesis. It departs from our general assumption that technical change only improves labor productivity growth while leaving capital productivity constant. The output-capital ratio will still be a constant in the long run, but as the result of the economy's dynamics rather than an assumption on the innovation technology.

Originated in the 1960s by Kennedy (1964) and von Weizsacker (1962), this theory has experienced a recent revival within the Classical-Marxian tradition (Foley, 2003; Julius, 2005; Rada, 2012; Tavani, 2012; Zamparelli , 2015; Rada et al., 2023). Its central element is the innovation possibility frontier (IPF), which inversely relates the attainable growth rate of labor productivity to the growth rate of capital productivity: $g_B = \epsilon(g_A), \epsilon' < 0, \epsilon'' < 0$. Firms choose the direction of technical change, that is a point (g_A, g_B) on the IPF, to maximize the growth rate of the profit rate: $\omega g_A + (1-\omega)g_B$. As a result, labor productivity growth becomes an increasing function of the wage share: $g_A = f(\omega), f' > 0$. The induced innovation theory thus provides a microfoundation for equation (10). However, in contrast with the microfoundation based on costly endogenous technical change, it does not assume constant capital productivity. This has dramatic implications on the nature of the long-run equilibrium. In fact, the long-run stability of the output-capital ratio determines the steady state wage share as solution to $g_B = \epsilon[f(\omega_{ss})] \equiv g_B(\omega_{ss}) = 0$, so that $\omega_{ss} = g_B^{-1}(0)$. At that point, the equality between warranted and natural growth rates is only responsible for the determination of the equilibrium capital productivity

$$sB_{ss}(1 - g_B^{-1}(0)) = n + f(g_B^{-1}(0)). \tag{19}$$

Accordingly, the saving rate only affects the output-capital ratio with no influence on long-run wage share. Still, a recent generalization of this framework to include costly innovation (Zamparelli , 2024) has clarified under what technological conditions the saving rate may affect the steady state income distribution.

4 Keynesian Growth

In contrast to Classical growth, Keynesian models rejects Say's law so that output is demand-determined. Within the Keynesian framework, we discuss three specifications of the investment function that corresponds to three different closures: a Kaleckian closure, where the rate of capacity utilization adjusts to ensure the equilibrium in the goods market; a Harrod-Kaldorian closure, where the saving-investment equilibrium occurs through changes in income distribution; and, finally, a Sraffian closure, where the investment to output ratio adjusts to ensures normal utilization rates despite exogenous income distribution. Multiple contributions (Cassetti, 2003; Palley, 1996; Sasaki, 2010; Allain , 2015; Lavoie, 2016; Taylor et al., 2019; Palley , 2019) may simultaneously belong to more than one of these categories, but this strict classification is a useful expositional device.

4.1 Kaleckian closure

In a basic Kaleckian model, firms operate with slack productive capacity and set prices (p) by charging a constant mark-up μ over unit labor costs: $p \equiv 1 = (1+\mu)\omega$, so that μ fully determines the labor share $\omega = 1/(1+\mu) = \bar{\omega}$. Therefore the Kaleckian distributive assumption is analogous to the classical closure with endogenous labor supply, but it is founded on firms' market power rather than on the tendency of real wages towards a socially determined subsistence level. Using (4) and $\omega = \bar{\omega}$ in the saving-investment equality we find the short-run equilibrium capacity utilization and growth rate as

$$u(\bar{\omega}, s) = \frac{\gamma_0 + \gamma_2(1-\bar{\omega})}{sB(1-\bar{\omega}) - \gamma_1},$$

$$g_K(\bar{\omega}, s) = \frac{sB(1-\bar{\omega})}{sB(1-\bar{\omega}) - \gamma_1}\left(\gamma_0 + \gamma_2(1-\bar{\omega})\right).$$

Capacity utilization, or aggregate demand, is wage-led and decreasing in the saving rate. Growth, however, is still a negative function of the saving rate, but can be either wage-led or profit-led depending on how strong the profitability effect on investment γ_2 is relative to the aggregate demand effect γ_1.

With endogenous labor supply the long-run equilibrium requires:

$$g_K(\bar{\omega}, s) = g_{L,ss} + g_A.$$

When growth is unconstrained by labor supply the short-run effects of income distribution and the saving rate fully translate to the long run, with employment growth operating as a buffer. Similarly to the Classical model with abundant labor, technical change determines employment growth given capital accumulation. When technical change is exogenous $g_{L,ss} = g_K(\bar{\omega}, s) - a$, which shows the trade-off between employment growth on one hand and productivity and real wage growth on the other hand. When we model productivity growth along Kaldor-Verdoorn lines, it retains the wage- or profit-led nature of capital accumulation $g_A = \varphi_0 + \varphi_1 g(\bar{\omega}, s)$, and so does employment growth $g_{L,ss} = g_K(\bar{\omega}, s)(1-\varphi_1) - \varphi_0$. Finally, with the Marxian motive we find

$g_{L,ss} = g(\bar{\omega}, s) - f(\bar{\omega})$. In this case, a higher wage share necessarily decreases employment growth when growth is profit-led, whereas when growth is wage-led the effect on employment will depend on whether the wage share positive shock raises accumulation more or less than productivity growth.

Things are particularly interesting when productivity growth is costly. If $R = \delta u BK$ the equilibrium utilization and growth rates turn into

$$u(\bar{\omega}, s, \delta) = \frac{\gamma_0 + \gamma_2(1 - \bar{\omega})}{B(s(1 - \bar{\omega}) - \delta) - \gamma_1}$$

$$g_K(\bar{\omega}, s, \delta) = \left(1 + \frac{\gamma_1}{B(s(1 - \bar{\omega}) - \delta) - \gamma_1}\right)(\gamma_0 + \gamma_2(1 - \bar{\omega})).$$

Capacity and growth are increasing in δ since R&D investment is a component of aggregate demand. Long-run productivity growth is $g_A = (\delta u(\bar{\omega}, s, \delta)B)^{\vartheta}$ and it has the short-run properties of capacity utilization. Thus, it is wage-led and decreasing in the saving rate. Additionally, short-run demand shocks, say changes in γ_0, have permanent effect on technical change. It is important to emphasize that, as long as demand is wage-led, technical change will be wage-led even when capital accumulation is profit-led because it is a mere function of output. Of course, things would change if we made R&D investment a function of both demand and profitability or, as we did in Section 3.2, if we assumed that it is a function of profits rather than output. If labor is abundant, in balanced growth we find

$$g_K(\bar{\omega}, s, \delta)) = g_{L,ss} + (\delta u(\bar{\omega}, s, \delta)B)^{\vartheta}.$$

The effect of changes in δ and s on employment growth are ambiguous since both accumulation and productivity growth move in the same direction. Positive shocks to ω necessarily reduce employment growth if capital accumulation is profit-led, while their effect is uncertain when growth is wage-led.

When labor supply is exogenous, the properties of the Kaleckian model change dramatically and we can no longer safely assume that income distribution is exogenous. If technical change is exogenous the long-run equilibrium condition becomes

$$g_K(\omega_{ss}, s) = n + a. \tag{20}$$

The right hand side is fully exogenous, so that the equality is achieved by adjustment in the wage share, which becomes the steady state endogenous variable. The effect of changes in a on income distribution will depend on the wage- or profit- led nature of the short-run growth rate as a higher or lower wage share will be needed to accommodate the rise in the natural growth rate $d\omega_{ss}/da = 1/g'_{\omega}$. On the other hand, a higher saving rate raises (lowers) the wage share if the economy is wage- (profit-) led: $d\omega_{ss}/ds = -g'_s/g'_{\omega} \gtrless 0 \iff g'_{\omega} \gtrless 0$. Since higher savings depress accumulation, income distribution must raise growth to preserve the equilibrium; hence the wage share will increase (decrease) when the economy is wage- (profit-) led.

When technical change evolves according to the Kaldor-Verdoorn law, income distribution is still the adjusting variable in the long-run, while the natural growth rate and productivity growth only depends on technological and demographic parameters:

$$g_K(\omega_{ss}, s) = \frac{n + \varphi_0}{(1 - \varphi_1)}, \quad g_{A,ss} = \frac{n\varphi_1 + \varphi_0}{(1 - \varphi_1)}.$$

If we let technical progress follow the classical-Marxian motive the balanced growth condition is:

$$g_K(\omega_{ss}, s) = n + f(\omega_{ss}). \tag{21}$$

Income distribution is still endogenous, and now the wage share directly influences the natural growth rate. Any factor affecting income distribution will have permanent effects on capital accumulation and labor productivity growth. For example, higher autonomous investments (γ_0) raise the wage share and growth when the economy is profit-led in the short-run since $d\omega_{ss}/d\gamma_0 = g_{\gamma_0}/(f_\omega - g_\omega) > 0$. When the economy is wage-led, faster accumulation will raise the wage share only if $f_\omega > g_\omega$, that is when the response of labor productivity growth to income distribution is stronger than the response of short-run accumulation.

When productivity growth react to the labor market tightness, we can acquire the employment rate as the adjusting variable in the long-run and return the wage share to its exogenous role

$$g_K(\bar{\omega}, s) = n + q(e_{ss}). \tag{22}$$

The employment rate and labor productivity growth will improve in line with capital accumulation. Positive shocks to the wage share will raise (lower) employment and technical change when growth is wage (profit-) led. They will also rise following a positive demand shock or a drop in the saving rate. Similar analyses can be found in Flaschel and Skott (2006); Lavoie (2006); Sasaki (2010).

Finally, income distribution becomes again endogenous when technical change is costly since the wage share is the only adjusting variable in the long-run equilibrium

$$g_K(\omega_{ss}, s, \delta) = n + (\delta u(\omega_{ss}, s, \delta)B)^{\vartheta}.$$

Productivity growth is endogenous and it responds to changes in the propensity to invest in R&D δ directly and indirectly through its effect on the wage share and capacity utilization. Productivity growth is a positive function of both δ and ω, but we cannot establish the overall effect of δ on the natural growth rate since the sign of $d\omega_{ss}/d\delta$ is ambiguous.

4.2 Harrod-Kaldorian closure

If we assume that capacity utilization adjusts to its long-run value, $u = u_n = 1$, the investment function (4) simplifies to $g_K = \gamma + \gamma_2(1 - \omega)$, with $\gamma \equiv \gamma_0 + \gamma_1$. The saving-investment equality is $sB(1 - \omega_{ss}) = \gamma + \gamma_2(1 - \omega_{ss})$, and determines the long-run profit share

$$(1 - \omega_{ss}) = \frac{\gamma}{sB - \gamma_2}.$$

The profit share is a positive function of autonomous investment as prices rise faster than wages in response to higher aggregate demand: investment 'forces saving' through distributional changes.

The corresponding growth rate is $g_K = \frac{sB\gamma}{sB-\gamma_2}$.

If labor supply is abundant, technical change has no effect on growth and distribution, but it determines how much capital accumulation is accommodated by employment or productivity growth. With exogenous technical change find $g_{L,ss} = \frac{sB\gamma}{sB-\gamma_2} - a$. Whereas when technical change is modeled an externality we have $g_{L,ss} = \frac{sB\gamma}{sB-\gamma_2}(1-\varphi_1) - \varphi_0$ in the Kaldor-Verdoon case, and $g_{L,ss} = \frac{sB\gamma}{sB-\gamma_2} - f(\frac{sB-\gamma_2-\gamma}{sB-\gamma_2})$ when we assume the Marxian motive. The only relevant difference between the last two cases is that productivity growth reacts in opposite way to changes in autonomous investment. It rises in the former, while it declines in the latter due to a fall in the wage share.

We now consider costly technical change. R&D investment is $R = \delta BK$ and affects the saving-investment equilibrium which becomes $sB(1-\omega_{ss}) = \gamma + \gamma_2(1-\omega_{ss}) + \delta B$. Since R&D investments contribute to aggregate demand the profit-share is a positive function of δ

$$(1-\omega_{ss}) = \frac{\gamma + \delta B}{sB - \gamma_2}.$$

The corresponding growth rate is also increasing in δ through its positive effect on profitability: $g_K = \frac{B(s\gamma+\gamma_2\delta)}{sB-\gamma_2}$. Labor productivity growth is $g_A = (\delta B)^{\vartheta}$ so that both accumulation and technical progress increase with δ, while the effect on employment will depend on which effect is stronger $g_{L,ss} = \frac{B(s\gamma+\gamma_2\delta)}{sB-\gamma_2} - (\delta B)^{\vartheta}$.

If we turn to exogenous labor supply, accumulation may be constrained by the natural growth rate, depending on the formalizations of technical progress. When $g_A = a$, long-run income distribution is anchored by technical change and population growth rather than accumulation:

$$sB(1-\omega_{ss}) = n + a.$$

Faster technical change reduces the wage share as it lowers labor demand. The interesting result is that the equality of actual and natural growth rate requires capital accumulation to lose its independent nature. In the long-run, not only income distribution but also autonomous investment must become endogenous to ensure equilibrium

$$\frac{sB\gamma_{ss}}{sB-\gamma_2} = sB(1-\omega_{ss}) = n + a.$$

In fact, aggregate demand plays no role and the model becomes indistinguishable from the Classical model with exogenous labor supply. The same is true when technical change follows the Kaldor-Verdoorn law:

$$\frac{sB\gamma_{ss}}{sB-\gamma_2} = sB(1-\omega_{ss}) = \frac{n+\varphi_0}{(1-\varphi_1)}.$$

In both cases, a higher saving rate increases autonomous investment and the wage share with no effect on growth. On the one hand, an increase in s reduces accumulation so γ_{ss} must rise to offset its effect; on the other hand, ω_{ss} increases as redistribution in favor of the class with the lower (zero) propensity to save is necessary to keep the warranted growth rate constant. The model can regain its Keynesian demand-led flavor if the parameters of the technical progress function become endogenous. Just as an example, if we assumed $\phi_1 = \phi_1(e)$,$\phi_1^{'} > 0$ (as in Setterfield, 2013), a demand shock would increase capital accumulation, labor demand, the employment rate and, finally, the natural growth rate.

When we follow the Marxian assumption, income distribution is still fixed by the equality between warranted and natural growth rates: $sB(1-\omega_{ss}) = n + f(\omega_{ss})$ and investment must adjust to satisfy the saving-investment equality: $\frac{sB\gamma_{ss}}{sB-\gamma_2} = sB(1-\omega_{ss})$. However, since the equilibrium wage share is a positive function of the saving rate, productivity growth is endogenous and increasing in s: $g_{A,ss} = f[\omega_{ss}(s)], g_A' > 0$. We also have a reversal of the paradox of thrift as capital accumulation must rise in line with the higher natural growth rate following the increase in s.

In contrast, the Keynesian character of the model can be fully restored when technical change responds to labor market tightness. Accumulation is exogenous once again and determines income distribution and productivity growth

$$\frac{sB\gamma}{sB-\gamma_2} = sB(1-\omega_{ss}) = n + q(e_{ss}).$$

Faster accumulation creates larger saving by reducing the wage share and faster productivity growth by increasing the employment rate through higher labor demand.

If we turn to costly innovation, long-run growth depends on the propensity to invest in R&D as the natural growth rate is $n + (\delta B)^{\vartheta}$. However, the Keynesian flavor of this closure is only partial since we cannot simultaneously retain the autonomous character of both R&D and physical capital investment. In fact, once δ pins down income distribution, $sB(1-\omega_{ss}) = n + (\delta B)^{\vartheta}$, the saving-investment equality requires γ to adjust endogenously $sB(1-\omega_{ss}) = \gamma_{ss} + \gamma_2(1-\omega_{ss})$.

4.3 Sraffian closure

A relatively recent and rapidly growing literature has used the notion of 'supermultiplier' to reconcile exogenous income distribution, normal capacity utilization, and demand-led growth (see Serrano 1995 for the seminal version of the model). The starting point is that investment is induced according to equation (5), but there is at least one component of aggregate demand, Z, which is autonomous, or independent of output, does not generate additional productive capacity and grows at the exogenous rate g_Z. Different types of expenditures, such as autonomous consumption, government expenditure, export, or residential investment have been considered plausible examples of autonomous demand. The crucial point is that the investment to output ratio in (5) is not a constant but adjusts to its steady state value h_{ss} to ensure that capacity utilization reaches its normal level $u = u_n = 1$. If income distribution is exogenous ($\omega = \bar{\omega}$), the steady state saving-investment equation yields

$$BK\left(s(1-\bar{\omega}) - h_{ss}\right) = Z, \tag{23}$$

which implies that in the long-run $g_K = g_Z$. By definition $g_K = I/K = h_{ss}B$, so that the exogenous rate of autonomous expenditure g_Z fixes h_{ss}. Going back to (23) and dividing both sides by K, we find

$$sB(1-\bar{\omega}) - g_Z = z_{ss},$$

where we defined $z \equiv Z/K$. Notice that higher autonomous demand growth reduces z_{ss} as in equilibrium it requires higher investment thus reducing the autonomous component of demand.

We now turn to the labor market and assume that labor supply is abundant. Once again, technical change has a passive role, it does not affect growth and distribution but residually determines employment growth given capital accumulation. When productivity growth is exogenous, we simply have $g_{L,ss} = g_z - a$. Similarly, when we assume the Kaldor-Verdoorn law $g_{L,ss} = g_Z(1-\varphi_1) - \varphi_0$, while $g_{L,ss} = g_Z - f(\bar{\omega})$ when technical change is induced by income distribution.

If productivity growth is costly, the saving-investment equilibrium is affected by R&D investment: $BK\left(s(1-\bar{\omega}) - h_{ss} - \delta\right) = Z$. Hence $g_Z = sB(1-\bar{\omega}) - \delta B - z_{ss} = (\delta B)^{\vartheta} + g_{L,ss}$. Growth is fully determined by the exogenous rate of growth of autonomous demand, while changes in R&D expenditure simply affects (in opposite way) the *level* of autonomous demand and employment growth. We could also think of R&D investment as the autonomous component of aggregate demand, that is $R = Z$. In this case, however, the level of R&D expenditure would not be an exogenous variable anymore as it would adjust to ensure the saving-investment equilibrium: $g_Z = sB(1-\bar{\omega}) - z_{ss} = {z_{ss}}^{\vartheta} + g_{L,ss}$.

When labor supply is exogenous, the demand-led properties of the model are compromised under some specifications of technical change. If productivity growth is exogenous, the natural growth rate is fully exogenous and the growth rate of autonomous demand thus loses its independent nature: $g_{Z,ss} = sB(1-\bar{\omega}) - z_{ss} = n + a$. The Kaldor-Verdoorn case is qualitatively identical as we have $g_{Z,ss} = sB(1-\bar{\omega}) - z_{ss} = \frac{n+\varphi_0}{(1-\varphi_1)}$. Things are more interesting if we assume the Marxian motive. If we keep distribution exogenous, g_Z still loses its independent character $g_{Z,ss} = sB(1-\bar{\omega}) - z_{ss} = n + f(\bar{\omega})$. Growth will be supply-side and wage-led. Positive shocks to the wage share raise the natural growth rate while demand and capital growth follow. We can re-establish the exogenous nature of g_z if we let income distribution be endogenous. Changes in g_Z will be accommodated by the wage share and the level of autonomous expenditure $g_Z = sB(1-\omega_{ss}) - z_{ss} = n + f(\omega_{ss})$; this solution is clearly closer in spirit to the Kaldorian closure than to the Sraffian one, but produces the unusual result of simultaneous positive correlation between aggregate demand growth and the labor share. When technical change reacts to the employment rate, g_Z regains its independent status even with exogenous distribution: $g_Z = sB(1-\bar{\omega}) - z_{ss} = n + q(e_{ss})$. Faster autonomous demand growth is enabled by a higher employment rate that raises the natural growth rate; at the same time, z_{ss} falls to make room for higher investment. Similar analyses are found in Fazzari et al. (2020); Palley (2019).

When productivity growth is costly and R&D investments are induced, only one between g_Z and the propensity to invest in R&D can be independent. If δ is an exogenous variable, growth is fully determined from the supply side and demand growth adjusts: $g_{Z,ss} = sB(1-\bar{\omega}) - z_{ss} = n + (\delta B)^{\vartheta}$. Nomaler et al. (2021) find an analogous result, though with a more complex productivity growth function that depends on the accumulated stock of R&D capital rather than on the flow of R&D investment. Otherwise, we need to assume that R&D investment would adapt to exogenous demand growth: $g_Z = sB(1-\bar{\omega}) - z_{ss} = n + (\delta_{ss} B)^{\vartheta}$. Finally, we face an analogous dilemma if we take R&D investment as autonomous expenditure. When we are free to choose R&D investment, demand growth is not independent anymore as it adjusts to the natural growth rate: $g_{Z,ss} = sB(1-\bar{\omega}) - z = n + z^{\vartheta}$. If, on the other hand, we choose to maintain the exogenous nature of demand growth, R&D investment is constrained in the long-run by demand: $g_Z = sB(1-\bar{\omega}) - z_{ss} = n + z_{ss}^{\vartheta}$.

5 Public R&D

We now turn to the public sector and consider the possibility that government expenditure affects the evolution of technology. While most heterodox research has focused on public physical investment and its complementarity with private capital (see Dutt 2013b; Tavani and Zamparelli 2016, 2017a), here we will study the impact of public R&D investment on labor productivity growth. Government collects taxes on profit income by charging the tax rate t. Taxes are fully spent in R&D investment so that $R = tuB(1-\omega)K$, and $g_A = (tuB(1-\omega))^{\vartheta}$.

In the Classical model with endogenous labor supply, the steady growth condition becomes

$$(1-t)sB(1-\bar{\omega}) = g_{L,ss} + (tB(1-\bar{\omega}))^{\vartheta}.$$

The tax rate plays a role analogous to the R&D investment share of saving δ, but it is now a policy instrument. Therefore, resolving the conflict between increasing labor productivity or employment growth becomes a government's prerogative. Given the wage share, raising t lowers capital accumulation and increases labor productivity growth, thus reducing employment growth. As $t \to 1$, growth stops while productivity growth reaches its maximum and employment falls (asymptotically to zero): $g_{L,ss} = -(tB(1-\bar{\omega}))^{\vartheta} < 0$.

When labor supply grows at a constant rate, the wage share becomes endogenous:

$$(1-t)sB(1-\omega_{ss}) = n + (tB(1-\omega_{ss}))^{\vartheta}.$$

The tax rate affects both income distribution and growth, so there will be a wage share- (t_ω) and growth rate- (t_g) maximizing tax rate. The relation between the two is not obvious in principle. In a more complex framework, Tavani and Zamparelli (2020) show that $t_\omega < t_g$ so that, on the one hand, policy-makers face a trade-off if they try to simultaneously maximize growth and reduce inequality, but, at the same time, for all $t < t_\omega$ raising the tax rate increases both growth and the wage share.

Turning to the Kaleckian model, public R&D investment is $R = tuB(1-\bar{\omega})$ so that the saving-investment equilibrium yields $(1-t)suB(1-\bar{\omega}) = \gamma_0 + \gamma_1 u + \gamma_2(1-\bar{\omega})$. Hence,

$$u(\bar{\omega}, s, t) = \frac{\gamma_0 + \gamma_2(1-\bar{\omega})}{(1-t)sB(1-\bar{\omega}) - \gamma_1}$$

$$g_K(\bar{\omega}, s, t) = \left(1 + \frac{\gamma_1}{(1-t)sB(1-\bar{\omega}) - \gamma_1}\right)(\gamma_0 + \gamma_2(1-\bar{\omega})),$$

with $du/dt, dg/dt > 0$. Both capacity utilization and capital accumulation increase in the tax rate because taxes are fully spent in R&D, which is a component of aggregate demand. With abundant labor supply, the balance growth condition now becomes

$$g_K(\bar{\omega}, s, t) = g_{L,ss} + (tu(\bar{\omega}, s, t)B(1 - \bar{\omega}))^{\vartheta}.$$

Productivity growth is also increasing in t both directly and through its positive effect on taxable income, while the effect on employment growth depends on the relative strength of the two positive effects on accumulation and technical change. On the other hand, if labor supply is exogenous the wage share becomes endogenous in steady state

$$g_K(\omega_{ss}, s, t) = n + (tu(\omega_{ss}, s, t)B(1 - \omega_{ss}))^{\vartheta}.$$

The policy-maker can manipulate the tax rate to affect income distribution and growth similarly to the Classical case; but the relation between t_{ω} and t_g is further complicate by the possibility that the economy may be wage or profit led.

In the Harrod-Kaldorian case, the equilibrium in the goods market determine the profit share as a positive function of the tax rate because higher taxes increase demand:

$$(1 - \omega_{ss}) = \frac{\gamma}{(1-t)sB - \gamma_2};$$

and since accumulation depends on profitability growth is also increasing in t

$$g = \frac{(1-t)sB\gamma}{(1-t)sB - \gamma_2}.$$

When we move to balanced growth with endogenous labor supply we find

$$\frac{(1-t)sB\gamma}{(1-t)sB - \gamma_2} = g_{L,ss} + \left(\frac{tB\gamma}{(1-t)sB - \gamma_2}\right)^{\vartheta}.$$

Productivity growth, $g_A = (tB(1-\omega_{ss}))^{\vartheta}$, increases in the tax rate both directly and through its positive effect on taxable income since t raises the profit share. Once again, the effect on employment is ambiguous. With exogenous labor supply we cannot simultaneously implement a discretionary fiscal policy while maintaining the exogenous nature of autonomous investment. Either one will have to go. For example, public investment in R&D determine private capital accumulation

$$\frac{(1-t)sB\gamma_{ss}}{(1-t)sB - \gamma_2} = n + \left(\frac{tB\gamma_{ss}}{(1-t)sB - \gamma_2}\right)^{\vartheta}.$$

In the Sraffian framework the saving-investment condition in steady state becomes

$$BK\left(s(1-t)(1-\bar{\omega}) - h_{ss}\right) = Z.$$

Given the exogenous growth rate of autonomous expenditure, z_{ss} is a negative function of the tax rate:

$$s(1-t)B(1-\bar{\omega}) - g_Z = z_{ss}.$$

Higher taxes means higher R&D, which means a lower level of autonomous expenditure if investment growth is given. If labor supply is endogenous, the tax rate determines productivity growth and employment growth with no effect on capital accumulation

$$g_Z = g_K = g_{L,ss} + \left(tB(1-\bar{\omega})\right)^{\vartheta}.$$

When labor supply is exogenous, on the other hand, we cannot simultaneously have exogenous distribution, exogenous growth of autonomous expenditure and independent choice of the tax rate. If growth is anchored by g_Z, changes in the tax rate necessarily affect income distribution:

$$g_Z = g_K = n + \left(tB(1-\omega_{ss})\right)^{\vartheta}.$$

6 Conclusion

This paper has reviewed alternative ways of modeling technical change within heterodox growth theories. We have formalized technological progress in three ways. First, we have simply taken it as exogenous and costless. Second, we have assumed that productivity growth occurs in the economy as an externality of other economic variables, namely capital accumulation, the wage share and the employment rate. Finally, we have posited that technical change is endogenous and costly because it requires private or public R&D investment.

We have coupled these formalizations of technical change with growth models classified by their different investment functions. We have started with a supply-side Classical model where investments coincide with full-capacity savings. We have then explored three models featuring Keynesian, demand-led, investment functions: a Kaleckian model with endogenous capacity utilization and exogenous income distribution; a Harrod-Kaldorian model with normal capacity utilization and endogenous income distribution; and a Sraffian model with both normal capacity utilization and exogenous distribution. All cases are studied under the alternative assumptions of inelastic or abundant labor supply.

We shall now try to summarize a few general lessons of our analysis. Overall, we have shown that changes in capital accumulation tend to have distributional effects with inelastic labor supply while they produce growth effects when the labor force is abundant; but the specific formalizations of technical change matter, particularly in labor constrained economies. When labor supply is abundant, and productivity growth is either exogenous or of the externality variety, technical change plays virtually no role in long-run growth and distribution outcomes. Whether capital accumulation be supply or demand determined, it is unconstrained by the labor market and fully determines long-run growth; while income distribution is either exogenous or decided in the goods market. Technical change merely determines the level of employment growth compatible with capital accumulation.

Productivity growth becomes more relevant when it is costly. Since R&D investments are part of aggregate demand, they will influence capacity utilization in the Kaleckian model and income distribution in the Harrod-Kaldorian model; changes in utilization and the wage share will, in turn, have permanent effects on growth.

The picture changes drastically and becomes more diverse when the economy is labor constrained. If technical change is exogenous or evolves along the Kaldor-Verdoorn lines, the long-run natural growth rate becomes a function of purely technological and demographic parameters. Both in the Classical and in Keynesian models, long-run growth is supply side and independent of saving or investment propensities. In the Classical, Kaleckian and Harrod-Kaldorian models capital accumulation adjusts to the natural growth rate through changes in income distribution; but while with the Kaleckian closure investment retains its independent exogenous nature, it becomes constrained by the natural growth rate in the Harrod-Kaldorian model. In fact, under these two formalizations of technical change, the Harrod-Kaldorian model is virtually identical to the Classical model in the long run, unless the parameters of the technical progress function are further assumed to be endogenous. With the Sraffian closure, autonomous demand growth rather than the wage share is the adjusting variable.

Things are more interesting when technical change follows the Marxian motive. Income distribution is still the adjusting variable in the Classical, Kaleckian and and Harrod-Kaldorian models, but long-run productivity and capital growth become sensitive to saving and investment preferences. The Classical and Harrod-Kaldorian closures retain their supply side nature but an increase in the saving rate raises the natural growth rate through its positive effect on the wage share. In the Kaleckian model, investment affects long-run productivity growth through its influence on the wage share, with the sign of the effect depending of whether the economy is wage- or profit- led and on the strength of the induced innovation effect. In the Sraffian model we obtain a wage-led supply side result if we keep income distribution exogenous, whereas long-run growth regain its demand-led nature if we let the wage share be the adjusting variable.

When productivity growth reacts to labor market tightness, the drivers of capital accumulation fully regain their ability to determine long-run growth. In the Classical model with exogenous wage shares, a positive shock to the saving rate increases capital accumulation, which is accommodated by higher employment and natural growth rates. The same occurs for demand shocks in the Kaleckian model, where, in addition, the employment productivity effect allows long-run growth to retain its wage- or profit- led short-run nature. In similar fashion, exogenous investment or autonomous demand growth are the ultimate sources of steady state growth when we assume, respectively, the Harrod-Kaldorian or Sraffian closures.

Next, we have turned to costly innovation with exogenous labor supply. We have obtained R&D investment as a profit maximizing choice only in the Classical model. The result is a microfoundation of the Marxian productivity function and, as a consequence, the wage share and the natural growth rate are increasing in the saving rate. In the Keynesian models, we have taken the propensity to invest in R&D as a given parameter. In the Kaleckian variant, both income distribution and long-run productivity growth are endogenous and depend on R&D investment, but we could not establish the sign of the two relations. In the Harrod-Kaldorian version of the model, R&D investments raise long-run growth but reduce the wage share, while investment in capital accumulation loses its exogenous nature. Similarly, growth of autonomous demand becomes endogenous to adjust to the R&D investment-determined natural growth rate under the Sraffian closure. The analysis of public R&D mostly resembles the discussion of private costly technical change, with the difference that the amount of R&D investment in the economy becomes a policy variable and can be used to target specific growth and distribution targets.

As parting thoughts, we should mention three subjects we did not address in our analysis. First, both in Classical (Michl and Tavani , 2022) and in Keynesian (Dutt, 2006, 2010) models, some of the technical change formalizations we discussed have been used to produce indeterminacy and path-dependence. The appeal of such possibilities for heterodox economics is apparent, but we have confined our discussion to unique steady state equilibria. Second, by assuming that workers do not save and accumulate wealth we have avoided to investigate the impact of technical change on class wealth inequality. This literature is still in a nascent stage (see Taylor et al. (2019); Cruz and Tavani (2023)) but appears destined to become ever more relevant in the context of increasing automation of production. Finally, heterodox economists have recently begun to introduce technical change in growth models concerned with climate mitigation (Taylor et al. , 2016; Naqvi and Stockhammer , 2018; Rezai et al. , 2018; de Oliveira, G. and Lima). The impact of technical change on environmental issues is beyond the scope of our investigation.

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