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Unconventional Policy and Idiosyncratic Risk*

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Abstract

Idiosyncratic uncertainty implies an additional channel that amplifies the transmission of persistent balance-sheet policies, through their effect on consumption risk. Through this channel, unconventional policy improves the central bank's ability to anchor private-sector expectations and to complement interest-rate policy in particular in response to deleveraging crises that expose the economy to the ELB on the policy rate. An application to the Great Financial Crisis suggests a key role of unconventional policy in managing the deleveraging cycle: while the natural interest rate is endogenous to private indebtedness—as in Benigno et al (2020)—the welfare-relevant target interest rate is not. The optimal policy response to debt deleveraging is in fact an unconventional one, it is associated to a shorter—rather than longer—optimal duration of zero interest-rate policies, and it does not involve front-loaded inflation during the ELB episode.

JEL codes: E21, E32, E44, E58
Keywords: Idiosyncratic risk; optimal monetary policy; unconventional policy; HANK; THANK; ELB; dynamic debt deleveraging.

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1 Introduction

Unconventional monetary policy and idiosyncratic uncertainty are arguably two of the most debated topics in macroeconomics in the past twenty years. Since the Great Financial Crisis of 2007-2009, most central banks in advanced economies have progressively increased the use of unconventional balance-sheet policies to overcome the limitations to conventional policy implied by the effective lower bound (ELB) on the nominal interest rate. The heavy use of such policies, when the cross-sectional impact of the financial crisis had increased economic inequality to record-high levels, stimulated a lively policy debate about the distributional implications of monetary policy, and the role of heterogeneity and idiosyncratic uncertainty.[1] On the academic side, while the empirical research has scrutinised both dimensions of policy,[2] the theoretical literature so far has mostly focused on the implications of idiosyncratic risk for the transmission of interest-rate policies.[3]

This paper contributes with a theoretical analysis of the interplay between unconventional balance-sheet policies and idiosyncratic uncertainty in a small-scale New Keynesian model with heterogeneous households and credit frictions. We build on the framework by Benigno et al. (2020), extended along the two dimensions referred to in the title: *i*) we introduce idiosyncratic uncertainty in the households' sector, in the form of labor-income risk and a stochastic transition between agent types, and *ii*) we allow financial intermediaries to hold interest-bearing central bank reserves, that can relax the borrowing constraint in the economy and makes unconventional policy effective.

In a simplified version of this extension, we show analytically the key role of idiosyncratic risk in the transmission of unconventional policy and leverage shocks, as well as in the central bank's ability to anchor private sector expectations. This simplified version also allows us to connect with the THANK literature spurred by the work of Bilbiie (2024), and discuss its implications for unconventional policy. In the general model, we then numerically study the economy's response to a deleveraging crisis. We show that, although the natural level of the interest rate is indeed endogenous to private indebtedness—as in Benigno et al. (2020)—it is also endogenous to unconventional policy; on the contrary, the welfare-relevant target level of the interest rate is independent of both. As a result, the optimal response to debt deleveraging is in fact an unconventional balance sheet expansion, associated with a lower (rather than higher) duration of zero interest rate policies (ZIRP).

In our model economy, households are either savers or borrowers. Savers smooth consumption over time by investing in short-term deposits; they work in the production sector and own both financial and non-financial firms. Borrowers, on average, are relatively poorer due to idiosyncratic labor income risk. They also work in the production sector and issue short-term bonds; in equilibrium, they have a higher marginal propensity to consume (MPC). Importantly, households only learn at the beginning of the period whether they are savers or borrowers, and cannot fully insure against the ensuing idiosyncratic income and consumption risk. A banking sector issues short-term

[1] See Yellen (2014), Bernanke (2015, 2017), and Schnabel (2021), among others.

[2] See Colciago et al. (2019) for a recent survey of the main empirical literature, discussing the lack of a general consensus on the effect of monetary policy—particularly along the unconventional dimension—on inequality.

[3] See Gornemann et al (2016), Kaplan et al (2018), Bilbiie (2018), Auclert (2019), and Acharya and Dogra (2020), among others. Two notable exceptions are Cui and Sterk (2021) and Sims et al (2022).

deposits to the savers and backs them with reserves from the central bank and private bonds from the borrowers, facing a leverage constraint that makes room for unconventional policy. The central bank issues reserves and purchases private bonds, besides controlling the short-term nominal interest rate, while the fiscal authority collects taxes and makes transfers on a balanced budget.

We show that idiosyncratic uncertainty plays a key role in shaping the transmission of expansionary balance-sheet policies to aggregate demand, in particular by mitigating the propagation and amplification of negative shocks through their effect on consumption risk.

In our economy, unconventional policy affects aggregate spending via three channels. The first, familiar one, works through the relaxation of the leverage constraint of financial intermediaries, which reduces borrowing costs and stimulates borrowers' consumption. Idiosyncratic risk and cyclical inequality imply two additional and novel channels. The "idiosyncratic-risk channel" in particular is key: by improving the outlook for borrowers, unconventional policy reduces consumption risk for savers which, in turn, find it optimal to reduce their precautionary savings and expand their current demand. This channel amplifies the expansionary effect of an unconventional monetary policy shock that initially only affects borrowers. In addition, via the "cyclical-inequality channel", a persistent increase in central bank's reserves further stimulates aggregate demand if consumption inequality is structurally counter-cyclical: the fall in expected inequality is amplified in general equilibrium by the expected boom in output, which reinforces the fall in consumption risk for savers and their incentive to increase current spending.

Via the idiosyncratic-risk channel, unconventional policy strengthens the central bank's ability to anchor private-sector expectations and rule out endogenous instability—even under an interest rate peg or in a permanent liquidity trap. This result is particularly significant, as it suggests that unconventional monetary policy enables the central bank to achieve price stability also in response to shocks that conventional policy cannot offset due to the ELB on nominal interest rates. However, we show that unconventional inflation targeting is not necessarily the optimal regime from a welfare perspective, as it may entail strong and persistent effects on consumption inequality that are detrimental to social welfare. The unconditionally optimal policy involves using conventional tools to pursue aggregate targets, and unconventional ones to address distributional objectives.

A numerical simulation of the general model—calibrated to interpret the deleveraging crisis that led to the Great Recession—suggests that the unconventional balance-sheet expansion by the Federal Reserve at the outset of the crisis may have had significant stabilizing effects on the output gap, inflation, and consumption inequality. Unconditional optimal policy requires that interest-rate cuts be complemented by balance-sheet expansions, which shortens the optimal duration of zero-interest rate policies. This element of the optimal policy reduces the relative appeal of front-loaded, above-target inflation compared to the RANK model—advocated by Benigno et al. (2020) on distributional grounds—because unconventional policy can address the cross-sectional effects of the ELB without relying on inflation.

Our analysis rationalizes the benefits of the recent evolution of central banking in advanced economies toward the "new-style" regime, in which both the conventional and unconventional tools are activated endogenously in response to the state of the economy.

This paper contributes mainly to two strands of the theoretical New-Keynesian literature.

The first one is the Heterogeneous Agents New Keynesian (HANK) literature, and in particular the *analytical* HANK literature.[4] With respect to this strand of the literature, and in particular with respect to Bilbiie (2018) which we follow in the tractable specification of heterogeneity and idiosyncratic risk, we contribute by introducing borrowing agents and credit frictions and by focusing on the unconventional dimension of monetary policy, whereas virtually all of the literature has so far focused on the conventional interest-rate policy.[5] Notable exceptions are Cui and Sterk (2021) and Sims et al (2022), which study the implications for unconventional policy of a prototypical *quantitative* HANK model. The former focus on the implications of different MPCs out of liquid versus illiquid assets, and find that, while balance-sheet policies can be highly stimulative, they also bear the welfare cost of potentially increasing inequality in the long run. The latter focus on credit frictions in an economy where the borrowing agents are wholesale firms financing purchases of physical capital by issuing long-term bonds, and find instead that the response of the economy to unconventional policy is essentially the same as in the RANK model. With respect to these papers, we exploit the tractability of our model to analytically characterise the additional theoretical transmission channels that idiosyncratic risk and cyclical inequality imply for balance-sheet policies.

We also contribute to the theoretical literature on unconventional monetary policy.[6] With the exception of the two papers above, most of this literature has ignored idiosyncratic uncertainty in the analysis of balance-sheet policies, regardless of whether the focus is on the aggregate effects in economies populated by a representative household (as e.g. in Gertler and Karadi, 2011 and 2013, Benigno and Nisticò, 2020, Karadi and Nakov, 2021, Benigno and Benigno, 2022, Bhattarai et al, 2022) or on the aggregate and distributive effects in economies populated by two types of heterogeneous households (as for example in Chen et al, 2012, Benigno and Nisticò, 2017, Del Negro et al, 2017, Sims et al, 2023, Bonciani and Oh, 2021, Wu and Xie, 2025). With respect to this literature, we contribute by studying the implications of idiosyncratic uncertainty, which allows us to identify additional transmission channels of balance-sheet policies related to consumption risk. Cúrdia and Woodford (2011, 2016) similarly study an economy where agents stochastically cycle between the borrowing and saving type, focusing on how this transition affects the dynamics and policy implications of credit spreads. With respect to these latter two papers, we study an economy

[4]The acronym is due to Kaplan et al (2018), initiating the *quantitative* HANK literature using frameworks with a rich household heterogeneity due to market incompleteness and generally require computationally demanding numerical methods to be solved. A non-exhaustive list of contributions in the *analytical* HANK literature, which instead uses tractable versions of the HANK model to scrutinise the theoretical channels and implications, includes Acharya and Dogra (2020), Acharya et al (2022), Bilbiie (2018, 2020), Challe (2020), Debortoli and Galí (2018, 2024), Ravn and Sterk (2018), Werning (2015). For a review of this literature, see Galí (2018); for a discussion of the relation between the HANK model and the corresponding Representative-Agent New-Keynesian (RANK) counterpart, see Kaplan and Violante (2018).

[5]Bilbiie et al (2022) develop an empirical version of Bilbiie (2018) and estimate it on US data to evaluate the role of cyclical inequality and idiosyncratic risk for business-cycle fluctuations.

[6]A non-exhaustive list of contributions in this strand includes Cúrdia and Woodford (2011), Gertler and Karadi (2011, 2013), Chen et al (2012), Benigno and Nisticò (2017, 2020), Del Negro et al (2017), Cui and Sterk (2021), Karadi and Nakov (2021), Sims et al (2022, 2023), Bonciani and Oh (2021), Bhattarai et al (2022), Benigno and Benigno (2022), Wu and Xie (2025).

where borrowing arises from idiosyncratic labor-income risk, rather than a shift in preferences, and we focus on the distributional dimension of balance-sheet policies and how it shapes the transmission mechanism to aggregate variables as well, pointing to consumption risk as a major channel.

The paper is organised as follows. Section 2 discusses the main novel features of our model economy with heterogeneous households, idiosyncratic risk and credit frictions. Section 3 studies analytically the positive and normative implications of idiosyncratic uncertainty in a simplified version of the model and discusses the relation with the THANK literature. Section 4 uses the general specification of the model to study the response of the economy to a deleveraging crisis and to discuss the role of unconventional policy and idiosyncratic risk. Section 5 concludes.

2 The Model Economy

To build our model economy, we move from the framework studied in Benigno et al. (2020) and extend it along the two dimensions that are referred to in the title: idiosyncratic risk and unconventional monetary policy. In this section we discuss these main additional features and their relevance for our perspective, and refer the reader to Appendix A for a detailed description of the full model.

The environment we borrow from Benigno et al. (2020) is populated by two types of households: a mass $1-z$ of savers, who accumulate financial wealth in terms of short-term bank deposits D and bank capital N, both yielding the nominal risk-free rate i^D, and a mass z of borrowers, who in equilibrium consume out of their debt B (on which they pay the nominal rate i^B) and disposable labor income. Financial flows between savers and borrowers are intermediated by a sector of banks facing a leverage constraint that limits the amount of lending they can provide to the private sector based on their net worth, in the spirit of Gertler and Karadi (2011, 2013).

To this framework, we first add idiosyncratic uncertainty, along two dimensions: i) a stochastic idiosyncratic employment status and ii) a stochastic transition between types. Each household j belonging to type $k = s, b$ is endowed with an idiosyncratic labor-market status $\varepsilon_{k,t}(j) = \{0, 1\}$, whereby their time-$t$ nominal labor income is

$$\mathcal{I}^L_{k,t}(j) \equiv \varepsilon_{k,t}(j) W_{k,t} L_{k,t}(j),$$

with $j \in [0, z]$ if $k = b$ and $j \in (z, 1]$ if $k = s$, and where W_k is the nominal wage for agents k and L_k their per-capita hours worked.[7] The idiosyncratic status $\varepsilon_{k,t}(j)$ is i.i.d. within each type, with

$$\text{prob}\left(\varepsilon_{k,t}(j) = 1 | k = b\right) = \varpi_t \quad < \quad \text{prob}\left(\varepsilon_{k,t}(j) = 1 | k = s\right) = 1,$$

where the probability of the high-income state for the borrowers is pro-cyclical: $\varpi_t = g(x_t)$, with $g(\cdot) \in [0, 1]$, $g_x(\cdot) > 0$ and $g(0) \equiv \varpi \in [0, 1]$, and where $x_t \equiv \log(Y_t/Y_t^*)$ denotes the log-output gap.

In addition, each period, an agent of type s faces the risk of becoming of type b, which oc-

[7] See McKay et al. (2016, 2017) and Ravn and Sterk (2021), among others, for a similar approach to modelling idiosyncratic employment and labor-income risk.

curs with constant probability $1 - p_s$, and a type-b agent the chance of becoming of type s, with constant probability $1 - p_b$.[8] For type-s agents, this transition means that they become exposed to unemployment risk and, thereby, that in equilibrium they have a relatively lower labor income. Ex-ante, this provides an incentive for a type-s agent to self-insure through precautionary saving. In contrast, for type-b agents, this transition means they can access a higher labor income. Ex-ante, this makes a type-b agent a "borrower", as it implies an incentive to engage in borrowing to make up for their lower income by anticipating a possible future switch to the s-type.[9] We will show that these "anticipative" motives are key for the transmission of balance-sheet policies.

To maximise tractability, we follow Bilbiie (2018) and Bilbiie et al (2022) and assume full insurance within each type but limited self-insurance across types: within each type agents fully share consumption, income and hours worked, while across types they only keep the one-period non-contingent securities in which they save or borrow. Savers can retain the assets through which they lend to banks in exchange for the risk-free rate i^D, but not those through which they lend to firms in exchange for their profits. This insurance mechanism implies that the savers' per-capita financial income includes the net payoff on non-contingent securities plus the profits of financial and non-financial firms, i.e. $\mathcal{I}^F_{s,t} \equiv p_s(1+i^D_{t-1})(D_{t-1}+N_{t-1}) - (1-p_s)(1+i^B_{t-1})B_{t-1} + (\Pi^f_t + \Pi^p_t)/(1-z)$, while for the set of borrowers it only includes the net payoff on non-contingent securities, implying $\mathcal{I}^F_{b,t} \equiv (1-p_b)(1+i^D_{t-1})(D_{t-1}+N_{t-1}) - p_b(1+i^B_{t-1})B_{t-1}$. As to labor income, full risk-sharing within the type implies $\mathcal{I}^L_{s,t} = W_{s,t}L_{s,t}$ for savers and $\mathcal{I}^L_{b,t} = \varpi_t W_{b,t}L_{b,t}$ for borrowers.

Optimal decisions imply the following Euler equations for savers and borrowers, respectively:

$$U_c(C_{s,t}) = \beta E_t \left\{ \frac{1+i^D_t}{1+\pi_{t+1}} \frac{\xi_{t+1}}{\xi_t} \Big[p_s U_c(C_{s,t+1}) + (1-p_s) U_c(C_{b,t+1}) \Big] \right\} \tag{1}$$

$$U_c(C_{b,t}) = \beta E_t \left\{ (1+\epsilon_t) \frac{1+i^B_t}{(1+\pi_{t+1})} \frac{\xi_{t+1}}{\xi_t} \Big[p_b U_c(C_{b,t+1}) + (1-p_b) U_c(C_{s,t+1}) \Big] \right\} \tag{2}$$

where C_k is per-capita consumption of agents of type k, U_c is the marginal utility of consumption, ξ is a preference shock on the discount factor β, π_{t+1} is the net inflation rate between period t and $t+1$ and ϵ_t is the elasticity of the credit spread to individual borrowing.

Equation (1) captures the main intuition we are going to build on: if borrowers are expected to consume less than savers, then a positive probability of turning borrower tomorrow ($p_s < 1$) makes a current saver want to hedge against the possible future drop in consumption by saving more today. If unconventional policy is able to improve the consumption outlook for borrowers, it also indirectly stimulates the current spending of savers, by reducing the consumption risk they face and thereby their precautionary savings. This is, in a nutshell, the idiosyncratic-risk channel of unconventional policy whose implications we analyse in the rest of the paper. Equation (2) emphasizes an additional implication of idiosyncratic risk: $p_b < 1$ activates "anticipative-borrowing" motives that affect current spending decisions of borrowers. If savers are expected to consume relatively more, a

[8] We impose the restriction $z(1-p_b) = (1-z)(1-p_s)$, which keeps the relative mass of the two types constant.

[9] Note that this also makes the assumption of different discount factors—typically adopted to induce a borrowing attitude—unnecessary in our economy. Henceforth, we will therefore focus on the case $\beta_k = \beta$, for $k = s, b$.

positive probability of turning saver tomorrow makes type-b agents want to anticipate the possible future rise in consumption by borrowing more and consume more also today.

The second feature we add makes room for unconventional policy: we allow the central bank to purchase private bonds B^c from the banking sector through open-market operations, using internal resources. The latter are equal to short-term nominal reserves R that bear the riskfree rate i^R and that the central bank can issue at will, plus any retained financial profit from the past. The central bank therefore faces the following flow-budget constraint

$$B_t^c = R_t + (1 + i_{t-1}^B)B_{t-1}^c - (1 + i_{t-1}^R)R_{t-1} - T_t^c,$$

where T^c are nominal remittances to the fiscal authority. The nominal reserves of the central bank define the unit of account in the economy. This implies that the central bank can independently choose three policy instruments: the interest rate on reserves i^R, the amount of reserves R, and the remittances T^c transferred to the treasury. As to the latter, we assume that the central bank remits its entire gross financial income each period t: $T_t^c = (1 + i_{t-1}^B)B_{t-1}^c - (1 + i_{t-1}^R)R_{t-1}$. This results in a constant zero level of nominal capital,[10] and a central bank balance sheet given by $B_t^c = R_t$.

The remaining two policy tools are the main objects of interest of our analysis, with the interest rate on reserves $i^R \geq 0$ capturing the *conventional* dimension of monetary policy, and the amount of reserves $R \geq 0$—i.e. the size of the central bank's balance sheet—the *unconventional* one.[11]

The interaction between the central bank and the intermediation sector has two consequences. First, the equilibrium interest rates on deposits and reserves are equalized by no-arbitrage: $i_t^D = i_t^R$. Second, the equilibrium aggregate amount—in real terms—of private debt, $b_t \equiv B_t/P_t$, is equal to the amount that meets the leverage constraint of the banks, $\theta_t n_t$ (where θ_t is the exogenous leverage ratio and $n_t \equiv N_t/P_t$ the banks' capital) plus the amount in the balance sheet of the central bank:[12]

$$b_t = \theta_t n_t + u_t, \tag{3}$$

where $u_t \equiv R_t/P_t$ denotes the central bank's reserves in real terms. By means of its unconventional tool, therefore, the central bank can ease the leverage constraint of the financial intermediaries and expand the borrowing capacity of the private sector and its spending decisions.

The model is completed by a fiscal authority and a continuum of non-financial monopolistic firms. The fiscal authority runs a balanced budget to impose taxes, provide redistributive transfers to households and employment subsidies to non-financial firms. The latter produce the differentiated goods using labor services and technology, and subject to nominal price rigidities, as in Benigno

[10]Since our analysis will be conducted in a first-order approximation of the model, we choose to disregard the implications of this assumption for the determination of the initial price level, which we take as predetermined. For a discussion, see Benigno (2020), Benigno and Nisticò (2025) and Benigno and Benigno (2022).

[11]As discussed in Benigno and Benigno (2022), the zero-lower bound on the interest rate on reserves needs not be an assumption, but rather an equilibrium outcome if the central bank issues also cash, that financial intermediaries can use as an alternative store of value in case reserves paid a negative interest rate. The economy remains cashless in equilibrium, but the existence of such an alternative store of value makes the zero-lower bound on i^R effective.

[12]For details on the aggregate equilibrium, please refer to Appendix A.5.

et al. (2020). As a result, the supply side is described by the familiar log-linear New-Keynesian Phillips Curve (NKPC):

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_{\varpi} x_t, \tag{4}$$

where $\hat{\pi}$ is the deviation of inflation from the target. Note that average marginal costs—reflecting the aggregate labor supply—are affected in our economy by the idiosyncratic labor-market status. If an increase in the output gap reduces employment risk, it stimulates a larger participation of borrowers in production and induces a downward pressure on marginal costs. This effect counteracts the familiar positive effect of the output gap on marginal costs, and is thus reflected in smaller slope of the NKPC, captured by

$$\kappa_{\varpi} \equiv \left(1 - \frac{z\varpi_x}{\sigma + \varphi}\right)\kappa$$

where $\kappa \equiv (\sigma + \varphi)\frac{(1-\vartheta)(1-\vartheta\beta)}{\vartheta}$ is the slope in the standard case, ϑ is the Calvo parameter, σ is the inverse intertemporal elasticity of substitution in consumption, φ the inverse Frisch elasticity of labor supply, and $\varpi_x \propto g_x(\,\cdot\,) > 0$ captures the degree of counter-cyclicality of unemployment risk.

We base our normative analysis on a linear-quadratic framework, whereby a second-order approximation of expected social welfare in our economy leads to the following quadratic loss function:[13]

$$\mathcal{L}_{t_0} = \frac{\sigma + \varphi}{2} E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(x_t^2 + \lambda_\pi \hat{\pi}_t^2 + \lambda_c \omega_t^2 + \lambda_l \left(\sigma\omega_t + \varpi_x x_t\right)^2 \right) \right\}, \tag{5}$$

with the relative welfare weights λ_i, for $i = \pi, c, l$, defined in Appendix C. Equation (5) identifies the sources of welfare loss, which in our economy comes from the familiar terms related to inflation and the output gap, and two additional terms, related to the cross-sectional inequality in consumption $\omega_t \equiv c_{s,t} - c_{b,t}$ and hours worked $l_{s,t} - l_{b,t} \propto \sigma\omega_t + \varpi_x x_t$.

An analogous role for the welfare costs of consumption (and hours) dispersion due to some kind of reduced-form households' heterogeneity arises in several other contributions in the literature, besides Benigno et al. (2020).[14] As in most of these contributions, the relative welfare weights on consumption and hours dispersion reflect the heterogeneity *between* the two agent types, while it is independent of the idiosyncratic uncertainty that generates stochastic transition between the types.[15] Unlike in this related literature, however, in our economy idiosyncratic risk does affect the welfare metrics, through the labor-market dimension. Equation (5) in fact generalises the loss function in Benigno et al. (2020)—where the inequality in hours worked is proportional to consumption dispersion only—to the case where it is also affected by the cyclicality of the idiosyncratic unemployment risk (ϖ_x), which implies an additional welfare role for output-gap fluctuations.[16]

[13] Under minor conditions, equation (5) is a valid second-order approximation of expected social welfare when evaluated using only first-order-approximated equilibrium conditions. Please refer to Appendix C for details.

[14] A non exhaustive list includes Benigno and Nisticò (2017), Bilbiie (2018), Bilbiie and Ragot (2021), Bonchi and Nisticò (2022), Nisticò (2016), Wu and Xie (2025) and, more indirectly, also Cùrdia and Woodford (2016).

[15] A notable exception, in the list above, are Nisticò (2016) and Bonchi and Nisticò (2022), where a different insurance mechanism makes the welfare-relevant consumption dispersion actually reflect the heterogeneity *within* the "saver" type, with the relative welfare weight thus critically depending on the transition probabilities.

[16] Equation (5) nests indeed the approximation of welfare in Benigno et al. (2020) when $\varpi_x = 0$.

Note that, whether or not the presence of consumption inequality in (5) implies for the optimal policy a meaningful departure from the standard RANK model depends however on the equilibrium drivers of consumption inequality. In the benchmark THANK model of Bilbiie (2018) equilibrium consumption inequality is proportional to the output gap, so that its presence in the welfare criterion results in a larger relative welfare weight on the latter, without altering the economic tradeoffs faced by policy. On the contrary, as we are going to show shortly, in our economy the credit friction implies a time-varying wedge between inequality and the output gap that responds to structural shocks.[17] Regardless of the shock hitting the economy, therefore, the role of consumption inequality for welfare cannot simply be reflected in a larger weight on output stabilisation and it always implies an additional and independent target with respect to inflation and output stabilisation.[18]

In particular, in our economy, the presence of consumption inequality in (5) implies for the *conventional* monetary policy an endogenous trade-off with inflation/output stability, despite the "divine coincidence" featured by the supply side. We will show that this endogenous trade-off can be resolved if the *unconventional* dimension of monetary policy is appropriately specified. Moreover, since idiosyncratic uncertainty affects the dynamics of this wedge, it will matter for the transmission of conventional and unconventional monetary-policy shocks, and for the way in which monetary policy optimally deals with the tradeoffs in the face of financial and real shocks.

3 Policy Implications: analytical results

We now introduce two simplifying assumptions that allow us to derive some clear analytical implications, which we later scrutinise numerically in the general model.

Assumption 1 *The banks are endowed with a constant real capital each period (i.e. $n_t = \bar{n}$).*

Assumption 2 *The set of borrowers receives each period a real transfer equal to:*

$$z\frac{T_{b,t}}{P_t} = \bar{T}_b + \nu\left(Y_t - \frac{W_t}{P_t}L_t\right) - \frac{\mathcal{I}^F_{b,t}}{P_t}. \tag{6}$$

Assumption 2 states that borrowers are partially bailed out, as implied by the last term in equation (6). Additionally, it specifies that they receive a share ν of the aggregate profits from non-financial firms and pay taxes to finance the same share ν of the employment subsidies (as in Benigno et al., 2020, and Bilbiie, 2018).

These assumptions, analogous but milder than those in Sims et al. (2023), have the appealing implication of eliminating the endogenous state variable related to private debt, allowing us to

[17] This is in general true also in other models where this kind of credit frictions gives rise to financial intermediation, like Benigno and Nisticò (2017), Benigno et al. (2020), Cùrdia and Woodford (2016).

[18] To be accurate, besides the additional and independent target, idiosyncratic uncertainty in our economy implies also a larger weight on the output gap compared to the RANK model, due to the cyclicality of employment risk ϖ_x.

derive clear analytical results.[19] In particular, the borrowers' budget constraint implies a static relation for their equilibrium consumption, which in a first-order approximation reads:[20]

$$c_{b,t} = \chi_{\varpi} x_t + \varpi y_t^* + z^{-1}(\hat{\theta}_t + \hat{u}_t). \tag{7}$$

A key parameter, in the equation above and in the rest of the analysis, is $\chi_{\varpi} \equiv \varpi\chi + \varpi_x \alpha$, with $\chi \equiv 1 + \left(1 - \frac{\nu}{z\varpi}\right)(\sigma + \varphi)$ and $\alpha \equiv \varpi(1-z) + \nu$. This parameter captures the borrower's MPC out of aggregate income. In the benchmark THANK model of Bilbiie (2018), an analogous parameter captures the MPC out of aggregate income of hand-to-mouth agents, and is key in the analysis of the role of inequality for the transmission of *conventional* interest-rate policy. As we will show, this parameter plays a crucial role also for *unconventional* balance-sheet policies. Importantly, in our economy the MPC of constrained agents depends not only on the fiscal redistribution parameter ν, but also on employment risk, both through its steady-state level ϖ and its cyclical component ϖ_x.

Proposition 1 *Suppose Assumptions 1 and 2 are satisfied, and that the savers' consumption in the steady state is higher than the borrowers' (i.e. $\bar{C}_s/\bar{C}_b > 1$). Then, in a first-order approximation, the equilibrium of the private sector is described by the following system of difference equations:*

$$x_t = E_t x_{t+1} - \sigma^{-1}(\hat{i}_t^R - E_t\hat{\pi}_{t+1} + E_t\Delta\hat{\xi}_{t+1}) + E_t\Delta y_{t+1}^* - \gamma E_t\omega_{t+1} - z\sigma^{-1}(\hat{i}_t^B - \hat{i}_t^R), \tag{8}$$

$$\omega_t = (\gamma_s + \gamma_b - 1)\, E_t\omega_{t+1} + \sigma^{-1}(\hat{i}_t^B - \hat{i}_t^R), \tag{9}$$

$$\omega_t = (1-z)^{-1}\left[(1-\chi_{\varpi})\, x_t + (1-\varpi)y_t^* - z^{-1}(\hat{\theta}_t + \hat{u}_t)\right], \tag{10}$$

$$\hat{b}_t = \hat{\theta}_t + \hat{u}_t, \tag{11}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa_{\varpi} x_t, \tag{12}$$

given the definitions $\gamma_s \equiv p_s/\Gamma_s$, $\gamma_b \equiv p_b/\Gamma_b$, and $\gamma \equiv (1-z)(1-\gamma_s) - z(1-\gamma_b)$, with $0 < \gamma_s, \gamma_b < 1$ since $\Gamma_s \equiv p_s + (1-p_s)U_c(\bar{C}_b)/U_c(\bar{C}_s)$ and $\Gamma_b \equiv p_b + (1-p_b)U_c(\bar{C}_s)/U_c(\bar{C}_b)$, which further implies $\gamma_s < p_s$, $\gamma_b > p_b$ and $\gamma > 0$, given $\Gamma \equiv \bar{C}_s/\bar{C}_b > 1$.

Proof. Please refer to Appendix B.1. ■

Equations (8)–(9) jointly characterize the IS schedule in our economy:

$$x_t = E_t x_{t+1} - \sigma^{-1}(\hat{i}_t^R - E_t\hat{\pi}_{t+1} + E_t\Delta\hat{\xi}_{t+1}) + E_t\Delta y_{t+1}^* + zE_t\Delta\omega_{t+1} - (1-\gamma_s)E_t\omega_{t+1}. \tag{13}$$

Equation (13) highlights the key implication of idiosyncratic uncertainty in our economy: if $\gamma_s < 1$, upward revisions in expectations about the level of future inequality trigger anticipative motives

[19] Assumption 2 is indeed milder than the one in Sims et al. (2023), which instead requires a *full* bail out to eliminate the state variable associated with debt. With idiosyncratic risk we only need a *partial* bail out for two reasons: *i*) only a share p_b of private debt remains in the hands of the borrowers in each period t, and *ii*) the latter can use the payoff from a share $1 - p_b$ of the assets of agents that were savers in $t-1$ (see the definition of $\mathcal{I}_{b,t}^F$).

[20] Throughout, we use the following definitions: $y_t \equiv \ln(Y_t/\bar{Y})$, $y_t^* \equiv \ln(Y_t^*/\bar{Y}^*)$, $c_{s,t} \equiv (C_{s,t} - \bar{C}_s)/\bar{Y}$, $c_{b,t} \equiv (C_{b,t} - \bar{C}_b)/\bar{Y}$, $\hat{b}_t \equiv (b_t - \bar{b})/\bar{Y}$, $\hat{u}_t \equiv (u_t - \bar{u})/\bar{Y}$, $\hat{\xi}_t \equiv \ln(\xi_t/\bar{\xi})$, $\hat{\pi}_t \equiv \pi_t - \pi^*$, and $\hat{i}_t^j \equiv i_t^j - \bar{\imath}^j$, for $j = B$, D, R, and $\hat{\theta}_t \equiv (\bar{b}_y - \bar{u}_y)\ln(\theta_t/\bar{\theta})$, with $\bar{b}_y \equiv \bar{b}/\bar{Y}$ and $\bar{u}_y \equiv \bar{u}/\bar{Y}$.

that act as a negative demand shock on aggregate output (last term in the equation). To our knowledge, this margin is novel in related environments with savers and borrowers, such as Benigno and Nisticò (2017), Benigno et al (2020) and Sims et al. (2023), and is instead familiar in the THANK literature spurred by the work of Bilbiie (2018). We will show that this margin implies two novel transmission channels of unconventional policy, compared to these related frameworks.

Equation (10) follows from the budget constraint of the borrowers, and it substantiates the claim stated in the previous section: the wedge between consumption inequality and the output gap is affected by potential-output shocks y_t^*, leverage shocks $\hat{\theta}_t$ and, importantly, changes in unconventional policy (i.e. $\hat{u}_t$). It also shows that if unemployment risk is more counter-cyclical (higher ϖ_x), so is consumption inequality (higher χ_ϖ). Given consumption inequality, equation (9) then determines the equilibrium credit spread, and equation (11) the evolution of private debt.

Once we specify appropriate rules for conventional and unconventional policy, the system (8)–(12) can determine the equilibrium value for the set of sequences $\{x_t, \omega_t, \hat{\pi}_t, \hat{b}_t, \hat{\imath}_t^B, \hat{\imath}_t^R, \hat{u}_t\}_{t=t_0}^{\infty}$, given the exogenous processes $\{y_t^*, \hat{\xi}_t, \hat{\theta}_t\}_{t=t_0}^{\infty}$.

3.1 Two Benchmark Allocations

To understand the policy implications and the relevant tradeoffs in our economy, note that the welfare-based loss function (5) implies a first-best allocation with $\hat{\pi}_t = x_t = \omega_t = 0$ for all t.

Consider first the "natural" allocation in a flexible-price equilibrium (denoted by ^{n}) where $\hat{u}_t = 0$, as in Benigno et al. (2020). The consumption-inequality equation (10) implies:

$$\omega_t^n = \frac{1-\varpi}{1-z} y_t^* - \frac{1}{z(1-z)} \hat{\theta}_t, \tag{14}$$

which, used in (8)–(9), yields the natural interest rate:

$$r_t^n = \sigma \frac{1-z\varpi}{1-z} E_t \Delta y_{t+1}^* - E_t \Delta \hat{\xi}_{t+1} - \frac{\sigma}{1-z} E_t \Delta \hat{\theta}_{t+1} - \sigma \frac{1-\gamma_s}{z(1-z)} E_t \left\{ z(1-\varpi) y_{t+1}^* - \hat{\theta}_{t+1} \right\}. \tag{15}$$

Note the following. First, the natural equilibrium may not be socially optimal if consumption inequality fluctuates—which occurs in response to efficient shocks to potential output (if there is unemployment risk, i.e. low ϖ) or to leverage shocks. If $\varpi = 1$ and $\hat{\theta}_t = 0$ for all t, the natural allocation is socially optimal, and coincides with the RANK model: $r_t^n = \sigma E_t \Delta y_{t+1}^* - E_t \Delta \hat{\xi}_{t+1}$. Second, deleveraging shocks lower the natural rate, as in Benigno et al (2020) and Eggertsson and Krugman (2012), challenging *conventional* policy at the ELB on nominal interest rates. Third, idiosyncratic uncertainty adds transmission channels. The natural rate over-accommodates potential-output shocks to offset consumption inequality pressures: the over-accommodation intensifies with *i*) higher uncertainty (lower γ_s), whereby a given change in inequality is more relevant for the output gap, and *ii*) greater labor-income risk (lower ϖ), whereby a given shock to potential output has larger effects on consumption inequality. In turn, to prevent the fluctuations in inequality from affecting the output gap, the natural rate also over-counteracts leverage shocks with respect to the case with no idiosyncratic risk (as, e.g., in Sims et al. 2023).

Moreover, we can establish the following

Proposition 2 *Suppose the economy satisfies the conditions of Proposition 1. Assume also that the central bank's reserves are held constant, i.e.* $\hat{u}_t = 0$ *for all* t. *Then, there is no stochastic process* $\{r_t\}_{t=t_0}^{\infty}$, *with* $r_t \equiv \hat{\imath}_t^R - E_t\hat{\pi}_{t+1}$, *consistent with the socially optimal allocation, in which* $\hat{\pi}_t = x_t = \omega_t = 0$ *for all* t *and for any vector of exogenous processes* $\{\mathbf{x}_t\}_{t=t_0}^{\infty}$ *with* $\mathbf{x}_t \equiv (\hat{\xi}_t,\, y_t^*,\, \hat{\theta}_t)$.

Proof. Imposing $\hat{\pi}_t = x_t = \omega_t = \hat{u}_t = 0$ for all t on the system (8)–(10) trivially implies that equation (10) is satisfied if and only if $\hat{\theta}_t = 0$ for all t and either $\varpi = 1$ or $y_t^* = 0$ for all t.

The real interest-rate path implied by (15) supports $\hat{\pi}_t = x_t = \hat{u}_t = 0$ for all t and for any vector of exogenous processes $\{\mathbf{x}_t\}_{t=t_0}^{\infty}$, but is inconsistent with $\omega_t = 0$ for all t, unless $\hat{\theta}_t = 0$ for all t and either $\varpi = 1$ or $y_t^* = 0$ for all t, as implied by (14). On the other hand, an interest-rate path can only be consistent with $\omega_t = 0$ for all t and for any vector of exogenous processes $\{\mathbf{x}_t\}_{t=t_0}^{\infty}$, if

$$x_t = (\chi - 1)^{-1}\left[(1-\varpi)y_t^* - z^{-1}\hat{\theta}_t\right] \tag{16}$$

for all t and any vector of exogenous processes $\{\mathbf{x}_t\}_{t=t_0}^{\infty}$, as implied by equation (10), that is if it induces fluctuations in the output gap that exactly offset the pressures on consumption inequality coming from potential-output or leverage shocks, and would therefore be inconsistent with $\hat{\pi}_t = x_t = 0$ for all t, unless $\hat{\theta}_t = 0$ for all t and either $\varpi = 1$ or $y_t^* = 0$ for all t. ■

Intuitively, if monetary policy only uses its *conventional* tool $\hat{\imath}_t^R$, the system lacks one degree of freedom to accommodate all three targets at once. Relaxing $\hat{u}_t = 0$ and letting the central bank's balance sheet adjust endogenously provides the degree of freedom that we need to reconcile stability of consumption inequality with the natural equilibrium:[21]

Proposition 3 *Suppose the economy satisfies the conditions of Proposition 1. Then, there exists a joint stochastic process* $\{r_t^*,\, u_t^*\}_{t=t_0}^{\infty}$, *with*

$$r_t^* = \sigma E_t\Delta y_{t+1}^* - E_t\Delta\hat{\xi}_{t+1}, \tag{17}$$

$$u_t^* = z(1-\varpi)y_t^* - \hat{\theta}_t \tag{18}$$

that is consistent with the first-best equilibrium (FBE) allocation, in which $r_t = r_t^*$, $\hat{u}_t = u_t^*$, *and* $\hat{\pi}_t = x_t = \omega_t = 0$ *for all* t *and for any vector of exogenous processes* $\{\mathbf{x}_t\}_{t=t_0}^{\infty}$ *with* $\mathbf{x}_t \equiv (\hat{\xi}_t,\, y_t^*,\, \hat{\theta}_t)$.

Proof. Imposing $x_t = \omega_t = 0$ for all t on equation (10) yields equation (18). Using the latter in equations (8)–(9), and imposing $\hat{\pi}_t = 0$ for all t, finally implies equation (17). ■

Propositions 2 and 3 jointly imply the following

Corollary 1 *Suppose the economy satisfies the conditions of Proposition 1. Then, an appropriate state-contingent path of the* **unconventional** *tool of monetary policy* $\hat{u}_t$ *is a necessary condition to support the socially optimal allocation.*

[21] For a result similar to Proposition 3 in an environment with no idiosyncratic uncertainty, see Wu and Xie (2025).

The intuition is simple: the conventional tool r_t^* addresses aggregate efficiency (where r_t^* is the same as in the RANK benchmark), while the unconventional tool $\hat{u}_t$ manages distributional concerns. Consequently, adjusting reserves can absorb leverage shocks without altering the real interest rate (as in Sims et al. 2023). More interesting and novel are the implications of fluctuations in y_t^*: as potential output expands, the optimal rate r_t^* falls to fully accommodate the increase in output—allowing savers to consume more by intertemporal substitution—while reserves rise to lower borrowing costs, and ensure that borrowers can benefit proportionately, compensating for the incomplete pass-through of potential output to labor income, due to employment risk ($\varpi < 1$). This coordinated response stabilizes consumption inequality at its optimal level, with the reserves adjustment being more pronounced when employment risk is higher (i.e. with lower ϖ).

3.2 The idiosyncratic-risk channel of unconventional policy

This Section characterizes the transmission of unconventional monetary policy to the real economy, and highlights the novel channels related to idiosyncratic consumption risk.

Using (18) and (17) into (10) and (13) allows us to reduce the IS schedule to:

$$x_t = \Phi E_t x_{t+1} - \sigma_x^{-1}(\hat{\imath}_t^R - E_t \hat{\pi}_{t+1} - r_t^*) - \delta E_t\{\Delta\hat{u}_{t+1} - \Delta u_{t+1}^*\} + z^{-1}\delta(1-\gamma_s)E_t\{\hat{u}_{t+1} - u_{t+1}^*\} \quad (19)$$

where $\sigma_x \equiv \frac{\sigma}{\delta(1-z)}$, $\delta \equiv (1 - z\chi_\varpi)^{-1}$ and $\Phi \equiv 1 + \delta(\chi_\varpi - 1)(1 - \gamma_s)$.[22] Equation (19) shows that unconventional monetary policy affects current aggregate demand via three channels:

First, the "borrowing-cost channel"—third term in (19). A current expansion in central bank's reserves stimulates demand because it relaxes intermediaries' leverage constraints, thus allowing borrowers to access cheaper credit and expand their consumption—see equation (7). This is the familiar transmission channel discussed in Gertler and Karadi (2011, 2013) and Sims et al. (2023).

Second, the "idiosyncratic-risk channel"—last term in (19). By boosting future borrowers' consumption, an expected expansion of the central bank's balance sheet reduces consumption risk for current savers. This, in turn, lowers precautionary savings and stimulates current aggregate demand. To our knowledge, this is a novel channel, that emphasizes the role of persistent unconventional policy in mitigating the shock propagation and amplification through consumption risk.

Third, the "cyclical-inequality channel"—first term in (19). If consumption inequality is countercyclical ($\chi_\varpi > 1$, implying $\Phi > 1$), a persistent increase in reserves amplifies the fall in expected future inequality via a larger output gap, further reducing consumption risk and enhancing the stimulative effect of the policy.[23] Bilbiie (2018) discusses how this mechanism affects conventional interest-rate policy. In our economy with credit frictions, equation (19) extends the implications of Bilbiie (2018) to the unconventional balance-sheet dimension of monetary policy as well.

To see this, and also to facilitate the intuition behind the result on determinacy we derive in the

[22]Note that for a large enough share of borrowers z the parameters δ and σ_x can turn negative (the "inverted aggregate demand logic" of Bilbiie, 2008). Henceforth, we restrict attention to the case $z\chi_\varpi < 1$, implying $\delta, \sigma_x > 0$.

[23]With procyclical inequality ($\chi_\varpi < 1$, implying $\Phi < 1$), this channel instead dampens the stimulative effect.

next section, note indeed that we can solve equation (19) forward and write the IS schedule as[24]

$$x_t = E_t \left\{ \sum_{k=0}^{\infty} \Phi^k \left[-\sigma_x^{-1} \left(\hat{\imath}^R_{t+k} - \hat{\pi}_{t+k+1} - r^*_{t+k} \right) \right. \right.$$

$$\left. \left. - \delta \left(\Delta \hat{u}_{t+k+1} - \Delta u^*_{t+k+1} \right) + z^{-1} \delta (1 - \gamma_s) \left(\hat{u}_{t+k+1} - u^*_{t+k+1} \right) \right] \right\}. \quad (20)$$

3.3 Equilibrium Determinacy

To study how monetary policy can steer the system toward the optimal equilibrium, we evaluate equilibrium determinacy under feedback rules for both conventional and unconventional tools.

Proposition 4 *Suppose the economy satisfies the conditions of Proposition 1. Assume also that the central bank sets conventional and unconventional policy according to the following feedback rules:*

$$\hat{\imath}^R_t = r^*_t + \phi_\pi \hat{\pi}_t + \phi_x x_t + v^c_t \quad (21)$$

$$\hat{u}_t = u^*_t - \psi_\pi \hat{\pi}_t - \psi_x x_t + v^u_t. \quad (22)$$

Then, a rational-expectations equilibrium is locally determinate if and only if the response coefficients in the feedback rules (21)–(22) satisfy the following inequality:

$$\sigma_x^{-1} \Big[(1-\beta)\phi_x + \kappa_\varpi (\phi_\pi - 1) \Big] + z^{-1} \delta (1 - \gamma_s) \Big[(1-\beta)\psi_x + \kappa_\varpi \psi_\pi \Big] > (1-\beta)(\Phi - 1). \quad (23)$$

Proof. Please refer to Appendix B.2. ■

Thus, the conventional and unconventional instruments are "perfect substitutes" for ensuring determinacy. Their substitution rate is given by the ratio of the output-gap elasticities with respect to the conventional interest-rate channel, σ_x^{-1}, and that with respect to the unconventional "idiosyncratic-risk channel", $z^{-1}\delta(1-\gamma_s)$. Note that this result follows directly from idiosyncratic uncertainty. In the polar case with $\gamma_s = \varpi_t = 1$ for all t (which implies $\varpi_x = 0$, $\kappa_\varpi = \kappa$, $\chi_\varpi = \chi$ and $\Phi = 1$ regardless of χ), where the "borrowing-cost channel" is still active because of the heterogeneity between savers and borrowers and the credit friction, the condition for determinacy reduces to $(1-\beta)\phi_x + \kappa(\phi_\pi - 1) > 0$—the same as in the RANK model (Bullard and Mitra, 2002). This clarifies that, in our economy, the key mechanism through which unconventional policy ensures equilibrium determinacy is the idiosyncratic-risk channel, rather than the more familiar borrowing-cost channel.

Note also the role of cyclical employment risk: when the latter is more counter-cyclical, local determinacy requires a stronger response (from either tool) because a larger ϖ_x both raises Φ and flattens the Phillips Curve, diminishing the policy's impact on inflation. In our framework, therefore, equilibrium determinacy is achieved by a convex combination of the responses from conventional and unconventional policy, rather than just from the conventional interest-rate policy.

[24] To be accurate, equation (20) also assumes that the effects of price stickiness vanish asymptotically (and in particular at a rate higher than Φ in the case $\Phi > 1$).

Corollary 2 *Suppose the economy satisfies the conditions of Proposition 1. Then, determinacy of the rational-expectations equilibrium can be achieved by means of unconventional tools only, i.e. even in the limiting case of an interest-rate peg (i.e. $\phi_\pi = \phi_x = 0$), or a permanent liquidity trap, as long as the unconventional policy is responsive enough, meaning it satisfies*

$$z^{-1}\delta(1-\gamma_s)\Big[(1-\beta)\psi_x + \kappa_\varpi \psi_\pi\Big] > (1-\beta)(\Phi - 1) + \sigma_x^{-1}\kappa_\varpi.$$

To grasp the power of this corollary, note that, in the special case where $\Phi = 1-(1-\beta)^{-1}\sigma_x^{-1}\kappa_\varpi$, the above condition further simplifies to $(1-\beta)\psi_x + \kappa_\varpi\psi_\pi > 0$: the central bank in this special case is able to rule out endogenous instability even if the only thing the private sector expects it to do is use its balance sheet to respond to inflation with a positive (however small) coefficient.

Finally, condition (23) generalizes another result derived for the conventional monetary policy by Bilbiie (2018): the cyclicality of consumption inequality determines the extent to which monetary policy—intended here as the *combination* of conventional and unconventional policy—needs to be active in order to implement equilibrium determinacy, with countercyclical inequality $(\chi_\varpi, \Phi > 1)$ requiring a higher degree of responsiveness to rule out sunspot fluctuations.[25] The interesting complementary insight that we provide is that a countercyclical inequality in our economy does not necessarily make the Taylor Principle insufficient for determinacy, as instead in Bilbiie (2018). Endogenous unconventional policy in fact improves the central bank's ability to anchor private-sector expectations, by providing the additional degree of responsiveness that is needed to rule out endogenous instability without deviating from the Taylor Principle, as long as

$$(1-\beta)\psi_x + \kappa_\varpi\psi_\pi > z(1-\beta)(\chi_\varpi - 1). \tag{24}$$

When inequality is counter-cyclical $(\chi_\varpi > 1)$ and there is idiosyncratic uncertainty $(\Phi > 1)$, an upward revision in income expectations overstimulates current demand, because of the amplification induced via the "cyclical-inequality channel" by the expectation that future inequality will fall. This challenges equilibrium determinacy unless monetary policy leans against the extra push on aggregate demand coming from a lower consumption risk for savers. There are two possible remedies to this: increase the responsiveness of interest-rate policy to inflation beyond the Taylor Principle, as shown by Bilbiie (2018), or use balance-sheet policy sufficiently actively, as implied by (24).

3.4 The Transmission of Shocks and the Role of Idiosyncratic Risk

This section exploits the tractability of our framework under Assumptions 1 and 2 to characterize analytically the effects of policy and non-policy shocks, study their transmission mechanism and the role of the idiosyncratic uncertainty and the cyclicality of inequality and unemployment risk.

[25] An analogous result is derived in Acharya and Dogra (2020), in a prototypical (though analytical) HANK model.

3.4.1 Unconventional policy shocks

Proposition 5 *Suppose the economy satisfies the conditions of Proposition 1. Assume also that the central bank follows the conventional policy rule (21), with ϕ_x, $\phi_\pi \geq 0$ satisfying the condition (23), and the unconventional policy rule (22), with $\psi_x = \psi_\pi = 0$, such that $\hat{u}_t = u_t^* + v_t^u$, where the unconventional policy shock follows an AR(1) process:*

$$v_t^u = \rho_u v_{t-1}^u + \varepsilon_t^u, \tag{25}$$

with $\rho_i \in [0, 1]$. Then, the equilibrium level of the output gap and inflation, conditional on an unconventional monetary policy shock, are, respectively

$$x_t = \sigma_x \delta \left[1 - \rho_u + \rho_u \frac{1 - \gamma_s}{z} \right] \Psi_u v_t^u \tag{26}$$

and

$$\hat{\pi}_t = \sigma_x \delta \frac{\kappa_\varpi}{1 - \beta \rho_u} \left[1 - \rho_u + \rho_u \frac{1 - \gamma_s}{z} \right] \Psi_u v_t^u \tag{27}$$

given the definition

$$\Psi_u \equiv \left[\sigma_x (1 - \Phi \rho_u) + \phi_x + \frac{\kappa_\varpi}{1 - \beta \rho_u} (\phi_\pi - \rho_u) \right]^{-1} \tag{28}$$

with $\Psi_u > 0$. Therefore:

1. *more idiosyncratic uncertainty (lower γ_s) makes unconventional policy more powerful*
2. *counter-cyclical unemployment risk ($\varpi_x > 0$) makes unconventional policy more effective on the output gap and less on inflation*
3. *a positive unconventional shock directly improves inequality; this effect is amplified if inequality is counter-cyclical ($\chi_\varpi > 1$), and dampened or even overturned if it is pro-cyclical ($\chi_\varpi < 1$).*

Proof. Using the method of undetermined coefficients with equations (12) and (19) yields (26) and (27). Differentiating these with respect to γ_s and ϖ_x gives the first two conclusions. Conclusion *3.* follows from substituting (26) and (27) in (10), which yields

$$\omega_t = -\frac{1}{z(1-z)} \left[1 + z \sigma_x \delta \Psi_u \left(1 - \rho_u + \rho_u \frac{1 - \gamma_s}{z} \right) (\chi_\varpi - 1) \right] v_t^u. \tag{29}$$

■

An expansionary unconventional policy unambiguously raises the output gap and inflation. Its expansionary effects transmit via the three channels outlined in Section 3.2. The "borrowing-cost channel" is captured by the first two terms in the square brackets of (26)–(27): a temporary ($\rho_u < 1$) increase in reserves lowers borrowing rates, boosting borrowers' consumption. The "idiosyncratic-risk channel" is captured by the last term in the square brackets of (26)–(27): a persistent ($\rho_u > 0$)

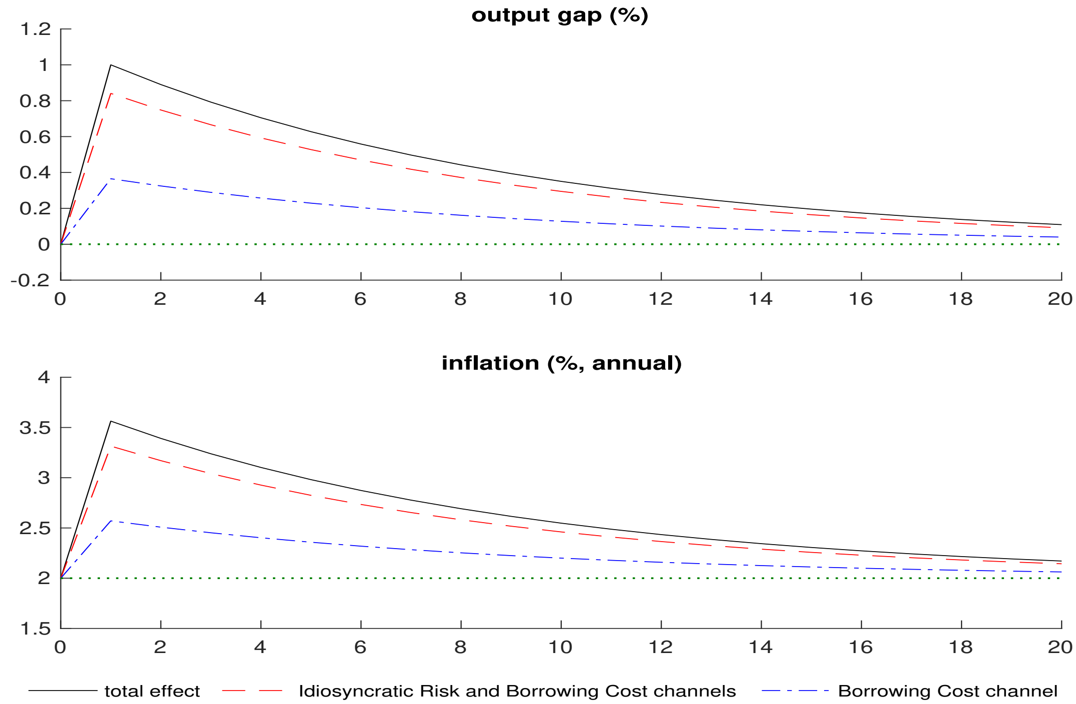

Figure 1: Isolating the transmission channels of an unconventional policy shock. Response when the sole "borrowing-cost channel" is active (blue dashed-dotted line); response when also the "idiosyncratic-risk channel" is active (red dashed line); response when also the "cyclical-inequality channel" is active (black solid line).

expansion reduces savers' consumption risk, providing an additional boost to aggregate demand. The "cyclical-inequality channel" finally affects coefficient Φ in definition (28): a counter-cyclical inequality ($\Phi > 1$) implies a general-equilibrium amplification through a larger Ψ_u. While the effect on demand is stronger when idiosyncratic uncertainty is larger (lower γ_s), more counter-cyclical employment risk (higher ϖ_x) reduces the effect on inflation, as it flattens the Phillips Curve.

Note that while the borrowing-cost channel is stronger for more transitory shocks (lower ρ_u), the idiosyncratic-risk channel becomes more effective as persistence increases. This highlights the importance of what we might call "unconventional forward guidance": the central bank must signal that its balance-sheet policies are persistent to increase its real impact and shape private-sector expectations effectively. Considering the very high persistence of the balance-sheet policies that we have observed in the past years, and the effort of central bankers in communicating such persistence, this result suggests a key role of the "idiosyncratic-risk channel" in the transmission of such policies.

To visualize the relative importance of the two additional channels implied in our economy compared to existing literature, Figure 1 displays a numerical illustration of the effects on the output gap and inflation of an increase in central bank's reserves that expands output by 1% on impact, with a half-life of about 6 quarters, for a benchmark calibration of the model.[26] The black solid lines in the figure show the responses when all channels are active, and where the idiosyncratic risk is calibrated to $p_s = 0.96$, which implies a value for $\Phi = 1.142$.[27] The red dashed lines show

[26]For the complete set of parameter values under the benchmark calibration, see Table 1 in Appendix E.

[27]The calibrated value for p_s is consistent with Bilbiie (2018), and with the estimated transition probabilities during downturns within the related framework of Bilbiie et al. (2022). We thank the authors for providing us with the

the case where we shut off the "cyclical-inequality channel", by forcing a value $\Phi = 1$, and the blue dashed-dotted lines show the case where we also shut off the "idiosyncratic-risk channel", by considering $\gamma_s = 1$ in the second term in square brackets in equations (26) and (27). The numerical illustration underscores the relevance of the "idiosyncratic-risk channel", which accounts for about half of the overall response, with roughly 15% explained by the "cyclical-inequality channel" and the remaining 35% by the traditional "borrowing-cost channel".[28]

Finally, unconventional policy affects inequality via two channels. One operates directly—first addendum in the square brackets in (29)—and reduces inequality through higher borrowing and higher borrowers' consumption, as also shown by equation (7). The other is indirect—second addendum in (29)—via general-equilibrium output adjustments. Their net impact depends on the cyclicality of inequality, with counter-cyclical inequality amplifying and pro-cyclical inequality dampening (or even reversing) the beneficial effect.

3.4.2 Deleveraging shocks

A deleveraging shock occurs when financial intermediaries' leverage constraints tighten, captured by an exogenous drop in $\hat{\theta}_t$, following the AR(1) process $\hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \varepsilon_t^\theta$, with $\rho_\theta \in [0, 1]$.

Proposition 6 *Suppose the economy satisfies the conditions of Proposition 1, and the central bank follows the policy rules (21) and (23), with ϕ_x, ϕ_π, ψ_x, $\psi_\pi \geq 0$ satisfying condition (23). Then, the equilibrium level of the output gap and inflation, conditional on a deleveraging shock, are, respectively*

$$x_t = \sigma_x \delta \left[1 - \rho_\theta + \rho_\theta \frac{1 - \gamma_s}{z} \right] \Psi_\theta \hat{\theta}_t \tag{30}$$

and

$$\hat{\pi}_t = \sigma_x \delta \frac{\kappa_\varpi}{1 - \beta \rho_\theta} \left[1 - \rho_\theta + \rho_\theta \frac{1 - \gamma_s}{z} \right] \Psi_\theta \hat{\theta}_t \tag{31}$$

given the definition

$$\Psi_\theta \equiv \left[\sigma_x (1 - \Phi \rho_\theta) + \varsigma_\theta \right]^{-1} \geq 0 \tag{32}$$

with

$$\varsigma_\theta \equiv \phi_x + \frac{\kappa_\varpi}{1 - \beta \rho_\theta} (\phi_\pi - \rho_\theta) + \sigma_x \delta \left(1 - \rho_\theta + \rho_\theta \frac{1 - \gamma_s}{z} \right) \left(\frac{\kappa_\varpi}{1 - \beta \rho_\theta} \psi_\pi + \psi_x \right) \tag{33}$$

capturing the overall degree of responsiveness of monetary policy. Therefore:

1. *more idiosyncratic uncertainty (lower γ_s) amplifies the effects of deleveraging shocks*
2. *counter-cyclical unemployment risk ($\varpi_x > 0$) amplifies the output response to the shock, while it dampens the inflation response*

smoothed estimates from their analysis.

[28] This result is in stark contrast with Sims et al. (2022), who find that the transmission of unconventional policy shocks is largely unaffected by idiosyncratic uncertainty in HANK economies. In their model, credit frictions in the banking sector impact the investment decisions of a wholesale non-financial firm rather than households' consumption choices, making idiosyncratic income risk naturally less relevant.

3. *a deleveraging shock increases inequality if the latter is counter-cyclical ($\chi_{\varpi} > 1$).*

Proof. The method of undetermined coefficients with (12) and (19) yields (30) and (31). Differentiating with respect to γ_s and ϖ_x implies conclusions *1.* and *2.* Using (30)–(31) in (10) yields

$$\omega_t = \frac{-\Psi_\theta}{z(1-z)}\left[z\sigma_x\delta\left(1-\rho_\theta+\rho_\theta\frac{1-\gamma_s}{z}\right)(\chi_{\varpi}-1)+(\sigma_x(1-\Phi\rho_\theta)+\phi_x)+\frac{\kappa_{\varpi}}{1-\beta\rho_\theta}(\phi_\pi-\rho_\theta)\right]\hat{\theta}_t, \quad (34)$$

trivially implying that a sufficient (though not necessary) condition for $\partial\omega/\partial\hat{\theta} < 0$ is $\chi_{\varpi} > 1$. ■

When the financial sector is forced to deleverage, the economy faces recessionary and deflationary pressures. Idiosyncratic risk ($\gamma_s < 1$) amplifies these effects via the partial-equilibrium increase in consumption inequality, which raises consumption risk for savers, who then raise their precautionary savings and cut their spending, as captured by the last term in the square brackets in (30)–(31).

Importantly, while the natural interest rate falls in response to the shock—as shown by equation (15)—the optimal interest rate remains unchanged—as shown by equation (17). This is an important implication, considering that most of the literature studying the monetary-policy response to a deleveraging crisis in the small-scale RANK model typically uses a fall in the natural rate as the primitive shock. Our analysis suggests that a drop in the natural rate does not directly guide the policy response, and that the monetary-policy regime plays an important role in the stabilization of a deleveraging shock. To see this, consider the general-equilibrium level of the policy rate under the conditions of Proposition 6:

$$\hat{\imath}_t^R = \sigma_x\delta\left(1-\rho_\theta+\rho_\theta\frac{1-\gamma_s}{z}\right)\left[\frac{\kappa_{\varpi}}{1-\beta\rho_\theta}\phi_\pi+\phi_x\right]\Psi_\theta\hat{\theta}_t. \quad (35)$$

A strong enough deleveraging shock is thus able to bring the policy rate to the ELB, exacerbating the recessionary and deflationary effects when the only policy tool is the conventional interest rate. In a policy regime that also activates the unconventional tools, instead, the general-equilibrium level of central bank's reserves responds to the shock according to

$$\hat{u}_t = -\sigma_x\delta\left(1-\rho_\theta+\rho_\theta\frac{1-\gamma_s}{z}\right)\left[\frac{\kappa_{\varpi}}{1-\beta\rho_\theta}\psi_\pi+\psi_x\right]\Psi_\theta\hat{\theta}_t. \quad (36)$$

To see how state-contingent unconventional policy can insulate the economy from the ELB and sterilize the shock, consider the case of an inflation-targeting central bank. Figure 2 provides a numerical illustration, and displays the response of our economy to a deleveraging shock bringing the natural interest rate to -5%, under three alternative regimes: a conventional inflation targeting (blue lines with dots), an unconventional inflation targeting (red lines with dots), and a conventional inflation targeting combined with an unconventional "inequality targeting" (green circled lines).[29]

Under a "conventional inflation-targeting" regime—the limiting case of $\phi_\pi \to \infty$—it is easy to

[29] Throughout, we perform the numerical simulations using the toolkit discussed in Eggertsson et al. (2021).

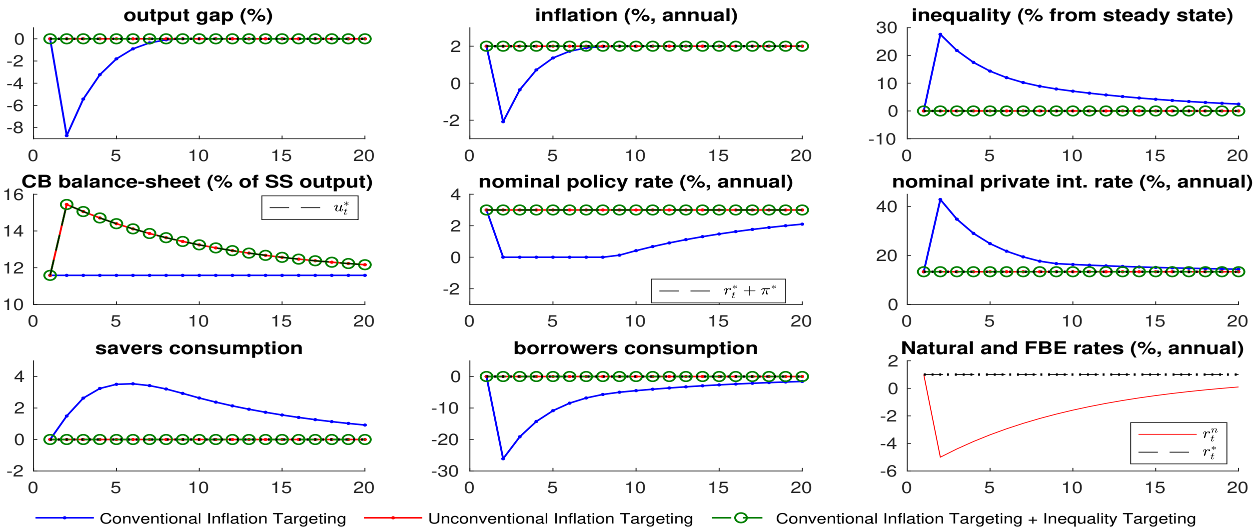

Figure 2: The response of the economy to a deleveraging shock. Blue lines with dots: conventional inflation targeting; red lines with dots: unconventional inflation targeting; circled green lines: conventional inflation targeting and unconventional inequality targeting.

show that, for given unconventional response coefficients ψ_π and ψ_x the following holds:

$$\lim_{\phi_\pi \to \infty} \Psi_\theta = 0 \qquad\qquad \lim_{\phi_\pi \to \infty} \left[\frac{\kappa_\varpi}{1-\beta\rho_\theta}\phi_\pi + \phi_x \right] \Psi_\theta = 1,$$

which implies

$$\hat{i}_t^R = \sigma_x \delta \left(1 - \rho_\theta + \rho_\theta \frac{1-\gamma_s}{z} \right) \hat{\theta}_t \qquad\qquad \hat{u}_t = 0. \tag{37}$$

As in the RANK model, a central bank using only the interest rate may be unable to hit the inflation target, as the ELB prevents the policy tool from tracking the natural rate when the latter goes negative. As shown in Figure 2, the policy rate stays at zero for about two years, resulting in a deep recession, deflation, and rising consumption inequality. Note that idiosyncratic uncertainty amplifies the recessionary and deflationary effects of the ELB: the persistent drop in borrowers' consumption raises consumption risk for savers, who increase their precautionary savings. This counteracts the expansionary effect for savers of the drop in the policy rate relative to the optimal one, preventing the increase on impact reported in related literature like Benigno et al (2020), and inducing an hump-shaped response instead.

Moreover, Proposition 3 suggests that (37) is inconsistent with the first-best allocation, as the optimal interest rate is independent of deleveraging shocks while the optimal level of reserves rises with them. Consider then the case of an "unconventional inflation-targeting" regime—i.e. the case $\psi_\pi \to \infty$. In this case, for a given level of conventional response coefficients ϕ_π and ϕ_x, it is easy to show that:

$$\lim_{\psi_\pi \to \infty} \Psi_\theta = 0 \qquad\qquad \lim_{\psi_\pi \to \infty} \left[\frac{\kappa_\varpi}{1-\beta\rho_\theta}\psi_\pi + \psi_x \right] \Psi_\theta = \left[\sigma_x \delta \left(1 - \rho_\theta + \rho_\theta \frac{1-\gamma_s}{z} \right) \right]^{-1},$$

which implies

$$\hat{\imath}_t^R = 0 \qquad\qquad \hat{u}_t = -\hat{\theta}_t. \tag{38}$$

The implied response of the interest rate and reserves is now consistent with Proposition 3. And indeed, the figure shows that this regime achieves full stabilization of the output gap, inflation, and consumption inequality at the same time. Appropriately adjusting central bank's reserves to hit the inflation target—where "appropriately" means so as to track the first-best level u_t^*, as shown in the middle-left panel—is able to completely absorb the effects of the leverage shock and prevent its pass through to consumption risk and aggregate demand.

This power of unconventional monetary policy follows from its distributional nature. To show this, Figure 2 displays the response under a policy regime combining conventional inflation targeting with a commitment to stabilize consumption inequality using central bank's reserves—i.e. when $\phi_\pi \to \infty$ in (21) and we replace (22) with

$$\hat{u}_t = \psi_\omega \omega_t + v_t^u \tag{39}$$

with $\psi_\omega \to \infty$. The figure shows that this regime is equivalent to unconventional inflation targeting: the commitment to stabilise consumption inequality is enough to achieve the first-best equilibrium, leaving the policy rate unchanged. Finally, equation (9) implies that the feedback rule (39) is equivalent to one where central bank's reserves are contingent on the expected future path of the credit spread: a commitment to use unconventional policy to permanently "close the spread" is able, in this economy, to implement the first-best allocation in response to deleveraging shocks.[30]

3.4.3 Discount-factor shocks

To further scrutinize our earlier claim that a fall in the natural rate does not dictate by itself the appropriate monetary-policy response, we examine the impact of a preference shock affecting discount factors, $\hat{\xi}_t$, which follows the AR(1) process $\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \varepsilon_t^\xi$, with $\rho_\xi \in [0, 1]$. This shock—commonly used in the RANK model to drive the natural rate negative and expose ELB issues—affects the reference interest rate in both the natural and first-best equilibria (see Section 3.1).

Proposition 7 *Suppose the economy satisfies the conditions of Proposition 1. Assume also that the central bank follows the policy rules (21) and (23), with ϕ_x, ϕ_π, ψ_x, $\psi_\pi \geq 0$ satisfying the condition (23). Then, the equilibrium level of the output gap and inflation, conditional on a discount-factor shock, are, respectively*

$$x_t = (1 - \rho_\xi)\, \Psi_\xi \hat{\xi}_t \tag{40}$$

and

$$\hat{\pi}_t = (1 - \rho_\xi)\, \frac{\kappa_\varpi}{1 - \beta\rho_\xi} \Psi_\xi \hat{\xi}_t \tag{41}$$

[30] For an analysis of the role of credit spreads in a related environment, see Cúrdia and Woodford (2011, 2016).

given the definition

$$\Psi_\xi \equiv \left[\sigma_x(1-\Phi\rho_\xi)+\varsigma_\xi\right]^{-1} \geq 0 \tag{42}$$

with

$$\varsigma_\xi \equiv \phi_x + \frac{\kappa_\varpi}{1-\beta\rho_\xi}(\phi_\pi-\rho_\xi)+\sigma_x\delta\left(1-\rho_\xi+\rho_\xi\frac{1-\gamma_s}{z}\right)\left(\frac{\kappa_\varpi}{1-\beta\rho_\xi}\psi_\pi+\psi_x\right), \tag{43}$$

capturing the overall degree of responsiveness of monetary policy. Therefore:

1. *idiosyncratic uncertainty ($\gamma_s < 1$) dampens the effects of preference shocks if unconventional policy follows an endogenous feedback rule (ψ_π, $\psi_x > 0$)*

2. *counter-cyclical unemployment risk ($\varpi_x > 0$) amplifies the output response to the shock, while it dampens the inflation response*

3. *a negative discount-factor shock increases inequality if and only if the latter is counter-cyclical ($\chi_\varpi > 1$) and the unconventional policy response is weak enough.*

Proof. Applying the method of undetermined coefficients to (12) and (19) yields equations (40) and (41). Differentiating with respect to γ_s and ϖ_x yields conclusions *1.* and *2.* Equation (43) in particular shows that either $\psi_\pi > 0$ or $\psi_x > 0$ is necessary for the derivative of (40) and (41) with respect to γ_s to be non zero. Conclusion *3.* follows from using (40) and (41) into (10), to yield:

$$\omega_t = \frac{1-\rho_\xi}{z(1-z)}\left[\psi_x+\frac{\kappa_\varpi}{1-\beta\rho_\xi}\psi_\pi - z(\chi_\varpi-1)\right]\Psi_\xi\hat{\xi}_t \tag{44}$$

which trivially implies that $\partial\omega/\partial\hat{\xi} < 0$ if and only if $\chi_\varpi > 1$ and $\psi_x + \frac{\kappa_\varpi}{1-\beta\rho_\xi}\psi_\pi < z(\chi_\varpi - 1)$. Since $\psi_\pi, \psi_x \geq 0$, pro-cyclical inequality ($\chi_\varpi < 1$) unambiguously implies $\partial\omega/\partial\hat{\xi} > 0$. ■

The equilibrium paths of the policy tools are

$$\hat{\imath}_t^R = (1-\rho_\xi)\left[\phi_x + \frac{\kappa_\varpi}{1-\beta\rho_\xi}\phi_\pi\right]\Psi_\xi\hat{\xi}_t \qquad \hat{u}_t = -(1-\rho_\xi)\left[\psi_x+\frac{\kappa_\varpi}{1-\beta\rho_\xi}\psi_\pi\right]\Psi_\xi\hat{\xi}_t.$$

As in the RANK model, a strong enough negative preference shock can force the nominal interest rate to the ELB if the policy rate responds to the shock ($\phi_x, \phi_\pi > 0$), amplifying the deflationary and recessionary effects. As in the case of a deleveraging shock, the unconventional tool offers an alternative to shield the economy from the effects of the ELB. To see this, consider first a conventional inflation-targeting regime ($\phi_\pi \to \infty$) and note that it implies

$$\lim_{\phi_\pi\to\infty}\Psi_\xi = 0 \qquad \lim_{\phi_\pi\to\infty}\left[\phi_x+\frac{\kappa_\varpi}{1-\beta\rho_\xi}\phi_\pi\right]\Psi_\xi = 1, \tag{45}$$

and thereby:

$$\hat{\imath}_t^R = (1-\rho_\xi)\hat{\xi}_t \qquad \hat{u}_t = 0. \tag{46}$$

By contrast, under an "unconventional inflation-targeting regime" ($\psi_\pi \to \infty$), one finds

$$\lim_{\psi_\pi \to \infty} \Psi_\xi = 0 \qquad \lim_{\psi_\pi \to \infty} \left[\frac{\kappa_\varpi}{1-\beta\rho_\xi}\psi_\pi + \psi_x \right] \Psi_\xi = \left[\sigma_x \delta \left(1 - \rho_\xi + \rho_\xi \frac{1-\gamma_s}{z} \right) \right]^{-1}, \quad (47)$$

which finally implies

$$\hat{\imath}_t^R = 0 \qquad \hat{u}_t = -(1-\rho_\xi) \left[\sigma_x \delta \left(1 - \rho_\xi + \rho_\xi \frac{1-\gamma_s}{z} \right) \right]^{-1} \hat{\xi}_t. \quad (48)$$

Therefore, while a conventional inflation-targeting regime may be unable to achieve price stability due to the ELB on nominal interest rates, *unconventional* inflation-targeting can fully sterilize even large negative shocks, thus insulating the economy from the ELB. Morevoer, if $\gamma_s < 1$, a given drop in $\hat{\xi}$ requires a smaller increase in central bank's liabilities to hit the inflation target, due to the enhanced power of unconventional policy with idiosyncratic uncertainty.

Figure 3 displays the response of the economy under the three regimes of Figure 2 to a discount-factor shock that brings the natural interest rate to -5%, as before, and provides several insights.

First, the path of the natural rate coincides with that of the first-best rate, as also implied by equations (15) and (17). This makes a zero interest-rate policy contractionary, due to the ELB. The path of the first-best level of central bank's reserves is instead flat at zero, as also implied by Proposition 3: the conventional interest rate is in principle the appropriate policy tool to respond to this shock. In the case of a small shock, this policy indeed achieves the socially optimal allocation.

Second, since the path of the natural rate is identical as in the case of a deleveraging shock—by construction—the aggregate response of output, inflation and the nominal interest rate under conventional inflation targeting is also identical. The transmission is however different. The cross-sectional distribution of the response is much milder, as captured by an increase in consumption inequality that is about 40% as large as in the case of a deleveraging shock (top-right panel). The contractionary stance of the interest-rate policy reduces savers consumption on impact, which dampens the anticipative-borrowing motive, the upward pressures on the private interest rate, and the drop in borrowers consumption. The weaker amplification through consumption risk is thus compensated by the stronger transmission via intertemporal substitution.

Third, an *unconventional* inflation-targeting regime allows the central bank to achieve price stability in spite of the ELB through the same balance-sheet expansion as in the previous section. However, while the output gap and inflation are stabilized, consumption inequality is not. To hit the inflation target, the central bank exploits the "idiosyncratic-risk channel" of unconventional policy to compensate for the inability to accommodate the intertemporal substitution called for by the preference shock—due to the ELB—and increases real reserves more than in the first-best equilibrium. This lowers the private interest rate, stimulates borrowers consumption and triggers a

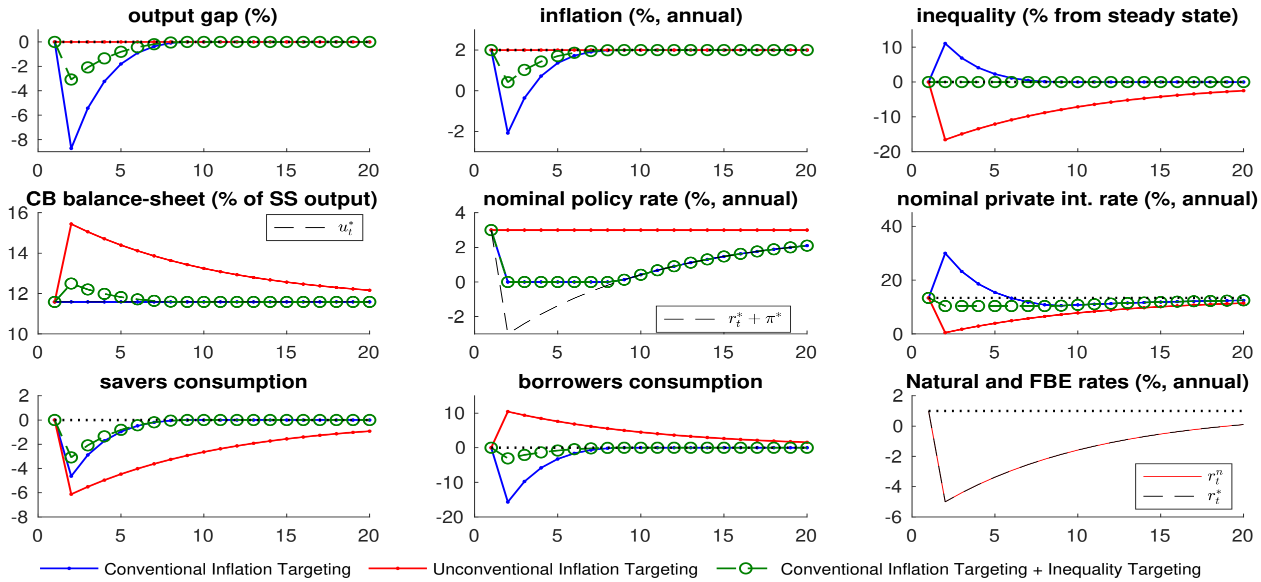

Figure 3: The response of the economy to a negative discount-factor shock. Blue lines with dots: conventional inflation targeting; red lines with dots: unconventional inflation targeting; circled green lines: conventional inflation targeting and unconventional inequality targeting.

strong and persistent decline in equilibrium consumption inequality, according to

$$\omega_t = \frac{1-\rho_\xi}{z(1-z)} \left[\sigma_x \delta \left(1 - \rho_\xi + \rho_\xi \frac{1-\gamma_s}{z} \right) \right]^{-1} \hat{\xi}_t. \tag{49}$$

Thus, while optimal for a dual-mandate central bank, the unconventional inflation-targeting regime is not from a welfare perspective, unlike in the case of a deleveraging shock. A central bank concerned with social welfare might want to use its conventional and unconventional tools selectively on aggregate and distributional targets. This is captured by the third regime in the figure, complementing the conventional inflation targeting with the unconventional inequality targeting. A mild increase in central bank's reserves in this case fully stabilizes consumption inequality and substantially improves on inflation and output-gap stabilization, compared to the conventional inflation-targeting regime. By stabilizing consumption inequality, the central bank mutes the idiosyncratic-risk channel that amplifies the effects of the ELB under conventional inflation targeting and accounts for about two thirds of the inflation and output gap volatility in that regime.[31]

Complementing the conventional inflation-targeting regime with an unconventional inequality targeting is therefore unambiguously welfare improving. However, the figure suggests that the central bank could use unconventional policy to raise welfare even more, by tolerating a small drop in consumption inequality in order to stabilize inflation and the output gap some more, through a lower consumption risk. In order to study this kind of policy tradeoffs more formally, in the next Section we turn to the analysis of optimal monetary policy in a linear-quadratic framework.

[31] The response of inflation and the output gap in this case coincides with the one in the RANK model.

3.5 Optimal Monetary Policy

The optimal policy minimizes loss (5) subject to (13), (10), (12) and the ELB on the policy rate:

$$\hat{\imath}_t^R \geq -\bar{\imath}^R. \tag{50}$$

Conditional on $\hat{u}_t = 0$ for all t, the optimal policy under discretion requires[32]

$$x_t + \lambda_l \varpi_x (\varpi_x x_t + \sigma \omega_t) = \kappa_\varpi \mu_{3,t} - \mu_{1,t} - \frac{\chi_\varpi - 1}{1 - z} \mu_{2,t} \tag{51}$$

$$\lambda_c \omega_t + \lambda_l \sigma (\varpi_x x_t + \sigma \omega_t) = -\mu_{2,t} - z \mu_{1,t} \tag{52}$$

$$\lambda_\pi \hat{\pi}_t = -\mu_{3,t} \tag{53}$$

$$\mu_{1,t} = -\sigma \mu_{4,t}, \tag{54}$$

where $\mu_{j,t}$, for $j = 1, ..., 4$ denote the multipliers respectively on (13), (10), (12), and (50). The above conditions yield the following targeting rule

$$\lambda_x^C x_t + \kappa_\varpi \lambda_\pi \hat{\pi}_t = \lambda_\omega^C \omega_t + \sigma_x \mu_{4,t}, \tag{55}$$

where we denote $\lambda_x^C \equiv 1 - \lambda_l \varpi_x \left[\sigma \frac{\chi_\varpi - 1}{1-z} - \varpi_x\right]$, $\lambda_\omega^C \equiv \lambda_\omega \frac{\chi_\varpi - 1}{1-z} - \sigma \lambda_l \varpi_x$ and $\lambda_\omega \equiv \lambda_c + \sigma^2 \lambda_l$.

Even if the ELB is not binding (i.e. $\mu_{4,t} = 0$), there is an endogenous tradeoff between inflation and output on the one hand, and consumption inequality on the other. Absent shocks with cross-sectional impact (i.e. $u_t^* = 0$) inequality is proportional to the output gap, as in Bilbiie (2018), and the tradeoff vanishes. In general, however, and in particular in response to leverage shocks, it is impossible to stabilize at the same time all three targets using only the conventional tool, and the central banks finds it optimal to induce some inflation/output volatility to reduce that in inequality.

In contrast, the unconditionally optimal policy (i.e. when also choosing the optimal path of reserves $\hat{u}_t$) requires the following additional condition:

$$\mu_{2,t} = 0. \tag{56}$$

When the ELB is not binding ($\mu_{1,t} = \mu_{4,t} = 0$) this implies that, with two policy tools, the central bank can hit two targets at once. Thus, the optimal targeting regime includes two rules:

$$x_t + \kappa_\varpi \lambda_{x\pi} \hat{\pi}_t = 0, \tag{57}$$

where we defined $\lambda_{x\pi} \equiv \frac{\lambda_\omega}{\lambda_\omega + \varpi_x^2 \lambda_l \lambda_c} \lambda_\pi < \lambda_\pi$, and

$$\lambda_\omega \omega_t + \sigma \varpi_x \lambda_l x_t = 0. \tag{58}$$

Rule (57) relates the two aggregate targets and informs the conventional dimension of the optimal

[32] Under discretion the "idiosyncratic-risk channel" working through anticipative motives is unexploited by definition. For the case of full commitment—which instead activates also this channel—see the next section.

policy, while (58) relates the two distributional targets—inequality in consumption and hours—and guides the unconventional dimension.

Note the role of idiosyncratic employment risk: if it is independent of the cycle (i.e. $\varpi_x = 0$), the targeting rule (57) becomes identical to the one implied by the RANK model,

$$x_t + \kappa \lambda_\pi \hat{\pi}_t = 0 \tag{59}$$

while (58) requires full stabilization of consumption inequality ($\omega_t = 0$ for each t), which is the only source of inequality in hours in this case. Thus, the optimal path for the policy rate is the same as in the RANK model, while the optimal path for the quantity of reserves follows the feedback rule

$$\hat{u}_t = u_t^* - z(\chi - 1)x_t, \tag{60}$$

where the response coefficient to the output gap is negative if $\chi > 1$.[33]

If employment risk is counter-cyclical ($\varpi_x > 0$), the welfare loss from output-gap fluctuations is amplified by the effects on the cross-sectional inequality in hours worked. In order to mitigate these distributional effects, equation (57) requires the optimal policy to tip the scale of the inflation-output tradeoff more in favor of the latter (i.e. $\lambda_{x\pi} < \lambda_\pi$) and equation (58) requires it to tolerate some fluctuations in consumption inequality. As a result, central bank's reserves respond less to the output gap in order to accommodate these optimal fluctuations in consumption inequality:

$$\hat{u}_t = u_t^* - z\left[\chi_\varpi - 1 - (1-z)\frac{\lambda_l \sigma \varpi_x}{\lambda_\omega}\right] x_t. \tag{61}$$

If the ELB is binding ($\mu_{4,t} > 0$), instead, the central bank can only pursue a single targeting rule, which solves the system (51)–(54) and (56):

$$x_t + \kappa_\varpi \lambda_\pi^{ELB} \hat{\pi}_t = z^{-1} \lambda_\omega^{ELB} \omega_t, \tag{62}$$

with $\lambda_\pi^{ELB} \equiv \frac{\lambda_\pi}{1 - z^{-1}\lambda_l \varpi_x(\sigma - z\varpi_x)}$ and $\lambda_\omega^{ELB} \equiv \frac{\lambda_\omega - \lambda_l z \varpi_x \sigma}{1 - z^{-1}\lambda_l \varpi_x(\sigma - z\varpi_x)}$. When the conventional tool is stuck at the ELB, the central bank should compensate with a stronger expansion in its balance sheet, to counteract the deflationary and recessionary effects of the shock that led the economy in the liquidity trap. We can see this by using (62) and (10) to solve for the optimal path of central bank's reserves:

$$\hat{u}_t = u_t^* - z\left[\chi_\varpi - 1 + \frac{z(1-z)}{\lambda_\omega^{ELB}}\right] x_t - z^2(1-z)\kappa_\varpi \frac{\lambda_\pi}{\lambda_\omega - \lambda_l z \varpi_x \sigma} \hat{\pi}_t, \tag{63}$$

which emphasizes a more aggressive responsiveness of unconventional policy both to the output gap and inflation, compared to (61).

[33]Note that in (59) and (60) we use $\kappa_\varpi = \kappa$ and $\chi_\varpi = \chi$, as implied by their respective definitions when $\varpi_x = 0$.

4 Policy Implications: the general model

To analyze the general model, we now relax assumptions 1 and 2, and replace them with:

Assumption 3 *Equity funding is endogenous, and subject to convex costs*

Assumption 4 *The set of borrowers receives each period real transfers equal to:*

$$z\frac{T_{b,t}}{P_t} = \bar{T}_b + \nu\left(Y_t - \frac{W_t}{P_t}L_t\right). \tag{64}$$

Assumption 3 implies that the solution of the bank's problem delivers an equilibrium credit spread that is directly related—as in Benigno et al. (2020)—to the amount of private debt in the economy and importantly—unlike in Benigno et al. (2020)—also to the central-bank balance sheet:

$$\frac{1+i_t^B}{1+i_t^R} = 1 + \mathcal{S}\left(\frac{b_t - u_t}{\bar{n}\theta_t}\right), \tag{65}$$

where $\mathcal{S}(\cdot)$ is a function of the convex costs of equity funding.[34] Equation (65) implies a direct channel of transmission of unconventional policy, whereby an expansionary policy (an increase in u_t) reduces the credit spread for a given conventional policy rate.

Assumption 4 in turn rules out any bailout of borrowers, who simply receive a share ν of the aggregate profits from non-financial firms and pay taxes to finance the same share ν of the optimal employment subsidy (as in Benigno et al., 2020). This assumption implies that the system of equations describing the equilibrium of the model includes the law of motion of private debt:

Proposition 8 *Suppose Assumptions 3 and 4 are satisfied, and that in the steady state the savers' consumption is higher than the borrowers' (i.e. $\Gamma > 1$). Then, in a first-order approximation, the equilibrium of the private sector is described by the following system of difference equations:*

$$\begin{aligned}
x_t &= E_t x_{t+1} - \sigma^{-1}(\hat{\imath}_t^R - E_t\hat{\pi}_{t+1} - r_t^*) - \gamma E_t\omega_{t+1} - z\sigma^{-1}\Big[\eta^{-1}\varrho(\hat{\imath}_t^B - \hat{\imath}_t^R) + v(\hat{b}_t - \bar{b}_u\hat{\theta}_t)\Big], && (66)\\
\omega_t &= (\gamma_s + \gamma_b - 1)\,E_t\omega_{t+1} + \sigma^{-1}\Big[\eta^{-1}\varrho(\hat{\imath}_t^B - \hat{\imath}_t^R) + v(\hat{b}_t - \bar{b}_u\hat{\theta}_t)\Big], && (67)\\
\hat{\imath}_t^B &= \hat{\imath}_t^R + \eta(\hat{b}_t - \hat{\theta}_t - \hat{u}_t), && (68)\\
\beta\hat{b}_t &= \left(\gamma_s + \gamma_b - \Gamma_s^{-1}\right)\Big[\hat{b}_{t-1} + \bar{b}_y(\hat{\imath}_{t-1}^R - \hat{\pi}_t)\Big] + \gamma_b\bar{b}_y(\hat{\imath}_{t-1}^B - \hat{\imath}_{t-1}^R)\\
&\quad + z\beta\Big[(1-\varpi)y_t^* - (\chi_\varpi - 1)x_t - (1-z)\omega_t\Big] && (69)\\
\hat{\pi}_t &= \beta E_t\hat{\pi}_{t+1} + \kappa_\varpi x_t, && (70)
\end{aligned}$$

given the parameters ϱ, η, $v > 0$, with $\varrho \geq \eta$, defined in Appendix B.3, the ratios $\bar{b}_y \equiv \bar{b}/\bar{Y}$ and $\bar{b}_u \equiv \bar{b}/\bar{b}^f \geq 1$—holding with strict inequality when $\bar{u} > 0$—and where $r_t^* \equiv \sigma E_t\Delta y_{t+1}^* - E_t\Delta\hat{\xi}_{t+1}$.

Proof. Please refer to Appendix B.3. ■

[34] For details, see Appendix A.1.

Once we specify appropriate rules for conventional and unconventional policy, the system (66)–(70) can determine the equilibrium value for the set of sequences $\{x_t, \omega_t, \hat{\pi}_t, \hat{b}_t, \hat{\imath}^B_t, \hat{\imath}^R_t, \hat{u}_t\}_{t=t_0}^{\infty}$, given the definition of r^*_t, the exogenous set of sequences $\{y^*_t, \hat{\xi}_t, \hat{\theta}_t\}_{t=t_0}^{\infty}$ and initial values $\hat{b}_{t_0-1}, \hat{\imath}^B_{t_0-1}, \hat{\imath}^R_{t_0-1}$.

Note the two differences with the simplified model of Proposition 1. The first is equation (68) replacing (11) and equation (69) instead of (10). The more general assumptions 3 and 4 imply that now the credit spread is pinned down by the equilibrium in the banking sector—equation (68)—the budget constraint of the borrowers determines the endogenous evolution of private debt—equation (69)—and the relative-Euler equation (67) determines the evolution of consumption inequality. The second difference, in equations (66)–(67), follows from a positive elasticity of the spread to individual borrowing. It implies a larger effect of the credit spread (given $\varrho \geq \eta$) and a direct effect of private debt in excess of the leverage ratio ($\hat{b}_t - \bar{b}_u\hat{\theta}_t$) on both the output gap and inequality dynamics.

Importantly, however, in spite of these differences, the IS schedule implied in the general model by (66)–(67) is again equation (13), and it thus characterizes the same role for expected inequality induced by idiosyncratic risk, from which all the implications analytically studied in Section 3 follow.

Let us now turn to the natural interest rate, that is central in the analysis of Benigno et al (2020):

Proposition 9 *Suppose the economy satisfies the conditions of Proposition 8, and consider a given central bank balance-sheet policy captured by a sequence $\{\hat{u}_t\}_{t=t_0}^{\infty}$. Then, the natural interest rate is*

$$r^n_t = r^*_t - z(\varrho + v)(\hat{b}_t - \hat{\theta}_t) + zv(\bar{b}_u - 1)\hat{\theta}_t + z\varrho\hat{u}_t - \gamma\sigma E_t\omega_{t+1}. \tag{71}$$

Proof. Equation (71) follows from using equation (68), and $x_t = \pi_t = 0$ for all t, in (66). ■

Equation (71) generalizes Benigno et al (2020) along three dimensions: *i*) idiosyncratic risk implies an additional channel through which private debt affects the natural rate, working through its effects on expected consumption inequality; *ii*) an unconventional balance sheet (i.e. $\bar{b}_u > 1$) amplifies the effects of leverage shocks on the natural rate beyond their transmission through the credit spread; *iii*) the natural rate is endogenous to unconventional policy as well as private debt.

From Proposition 9 it follows that the implications of Proposition 2 extend to the general model: when the conventional one is the only policy tool available ($\hat{u}_t = 0$ for all t), there is no path for the real interest rate consistent with the first-best allocation, as the evolution of debt or leverage shocks always induce—through the credit spread—fluctuations in consumption inequality that are detrimental for welfare, even when output and inflation are fully stabilized. Indeed, with stable output gap and inflation, if the real rate tracks (71) for all t, consumption inequality follows:

$$\omega^n_t = z^{-1}(\gamma_s + z - 1)E_t\omega^n_{t+1} - z^{-1}\sigma^{-1}(r^n_t - r^*_t),$$

which is generally different from zero whenever there are fluctuations in private debt or leverage ratios that make $r^n_t \neq r^*_t$.

Therefore, while the natural rate is indeed the real interest rate consistent with price stability and a stable output gap, it is not the one a central bank concerned with social welfare should like to achieve. While this result holds true also for $\gamma_s = 1$ and $\gamma = 0$, idiosyncratic risk (i.e. $\gamma_s < 1$ and

$\gamma > 0$) makes matters worse, as a persistent deleveraging shock tends to raise expected inequality, which in turn lowers the level of the natural rate more. As in the simplified model, considering the unconventional dimension of policy helps clarifying this point, with a revision of Proposition 3:

Proposition 10 *Suppose the economy satisfies the conditions of Proposition 8. Then, there exists a joint stochastic process* $\{r_t^*,\, u_t^*\}_{t=t_0}^{\infty}$, *with*

$$r_t^* = \sigma E_t \Delta y_{t+1}^* - E_t \Delta \hat{\xi}_{t+1}, \tag{72}$$

$$u_t^* = \frac{\varrho + \upsilon}{\varrho}\left(\hat{b}_t - \hat{\theta}_t\right) - \frac{(\bar{b}_u - 1)\upsilon}{\varrho}\hat{\theta}_t \tag{73}$$

that is consistent with the first-best equilibrium (FBE) allocation, in which $r_t = r_t^*$, $\hat{u}_t = u_t^*$, *and* $\hat{\pi}_t = x_t = \omega_t = 0$ *for all* $t \geq t_0$, *given initial conditions* $\hat{b}_{t_0-1}, \hat{\imath}^B_{t_0-1}, \hat{\imath}^R_{t_0-1}$ *and for any vector of exogenous processes* $\{\mathbf{x}_t\}_{t=t_0}^{\infty}$ *with* $\mathbf{x}_t \equiv (\hat{\xi}_t,\, y_t^*,\, \hat{\theta}_t)$.

In this equilibrium, $r_t^n = r_t^*$ *for all* t, *and the credit spread is not necessarily zero, as it follows*

$$\hat{\imath}_t^B - \hat{\imath}_t^R = \eta \frac{\upsilon}{\varrho}\left(\bar{b}_u \hat{\theta}_t - \hat{b}_t\right). \tag{74}$$

Proof. Imposing $\omega_t = 0$ for all $t \geq t_0$ on equation (67) implies that in the first-best equilibrium the credit spread follows (74). Using this restriction on equation (68) then implies (73). Imposing $\omega_t = x_t = \hat{\pi}_t = 0$ and equation (74) for all $t \geq t_0$ in equation (66), finally implies equation (72). ■

Intuitively, since fluctuations in private debt and the credit spread affect consumption inequality, the real interest rate does not need to absorb them as long as the central bank's reserves adjust to steer the banking sector's equilibrium toward an appropriate dynamics of the credit spread that stabilizes inequality. Proposition 10 has profound and meaningful implications.

Corollary 3 *Suppose the economy satisfies the conditions of Proposition 8. Then, from a welfare perspective, what is endogenous to private indebtedness is not the target level of the real interest rate, but rather the target level of real central bank's reserves. Consequently, an appropriate state-contingent path of the* **unconventional** *tool of monetary policy* $\hat{u}_t$ *is necessary to support the socially optimal allocation in response to fluctuations in private debt or financial leverage.*

The above corollary is key, because it suggests that the optimal monetary-policy response to debt deleveraging is in fact not a conventional interest-rate cut, as in the economy analyzed by Benigno et al (2020), but rather an unconventional expansion in the central bank's balance sheet. The intuition is straightforward: since it is transmitted through variations in the credit spread, a deleveraging shock should be dealt with using the policy tool that directly affects the determinants of such spread, i.e. unconventional policy. Furthermore, this clarifies that the nature of unconventional balance-sheet policy is intrinsically distributional—as shown by equation (67)—and it suggests that it can optimally complement the conventional interest rate policy in pursuing a welfare objective that includes both aggregate and distributional targets.

4.1 Unconventional policy and idiosyncratic risk in the Great Financial Crisis

Here we provide a numerical illustration of the mechanisms discussed above in a calibrated version of the general model that is meant to capture the state of the US economy at the outset of the Great Financial Crisis, before unconventional policy measures were introduced.[35] Our baseline policy scenario assumes that, when the crisis hits, the central bank seeks to stabilize inflation using its conventional interest-rate tool, and it complements the zero interest-rate policy with a permanent unconventional balance-sheet expansion worth about 6% of GDP.[36] To illustrate the relative role of conventional versus unconventional policies, we then calibrate a combination of discount-factor and leverage shocks—as well as the value of parameters η and ϱ—that produces in this baseline policy scenario a dynamic response of private debt and borrowers' interest rate that are consistent with the observed ones during the Great Recession.[37] The dynamic implications of this calibration strategy is displayed in Figure 7 in Appendix E, which shows that the model is able to account for a response of the output gap and inflation that is broadly in line with the data.

Figure 4 then shows the simulated response of the economy to the crisis under three alternative monetary-policy scenarios. Besides the baseline scenario (solid blue line), the figure shows the two opposite polar cases regarding unconventional policy: *i*) a central bank using only the interest-rate policy to implement inflation targeting (as in Benigno et al, 2020, red line with dots), and *ii*) a central bank using the balance-sheet policy to complement a conventional inflation targeting with an "unconventional inequality targeting" implying $\hat{u}_t = u_t^*$ for all t (green dashed line).

The implications shown in the figure are qualitatively consistent with those of the simplified model and offer additional insights into the analysis of the deleveraging crisis discussed elsewhere in the literature. In particular, the numerical illustration highlights the substantial benefits of incorporating an unconventional component into the policy response. When the central bank relies solely on its conventional interest-rate policy, the recessionary and deflationary effects—and the accompanying increase in consumption inequality—are about twice as severe. Furthermore, the figure confirms another finding discussed in Section 3.4: exploiting the distributional nature of unconventional policy to target consumption inequality (or the credit spread) further improves the dynamic response of the economy, and can promote a faster exit from the liquidity trap.

The role of idiosyncratic risk can be appreciated by examining the relative response of savers' consumption under the three policy regimes. Compared to the two regimes using unconventional policy, under "conventional inflation targeting" the interest-rate policy is relatively less restrictive in the early quarters due to the longer expected duration of the ZIRP. Nevertheless, savers' con-

[35] Specifically, we calibrate the steady state so as to match the state of the US economy in 2008q4.

[36] This is the size of the first round of unconventional measures, announced in November 2018 and March 2009.

[37] As the empirical counterpart of b_t we use the stock of debt of the private non-financial sector of the US economy, net of residential mortgages. See Table 1 for a detailed reference to the data. We use private debt net of mortgages because in our model debt is sustained in order to bring consumption forward, and not to acquire an asset (which is instead the case of mortgage debt). See also Curdia and Woodford (2016). Figure 6 in Appendix E shows that total and non-mortgage private debt evolved in a very similar way until the early 2000's, when the buildup of debt and subsequent permanent deleverage mostly involved mortgages, due to the housing bubble. Private non-mortgage debt displays instead a deleveraging process that is both milder and more transitory, with the stock of debt returning to its pre-crisis level by the end of 2014.

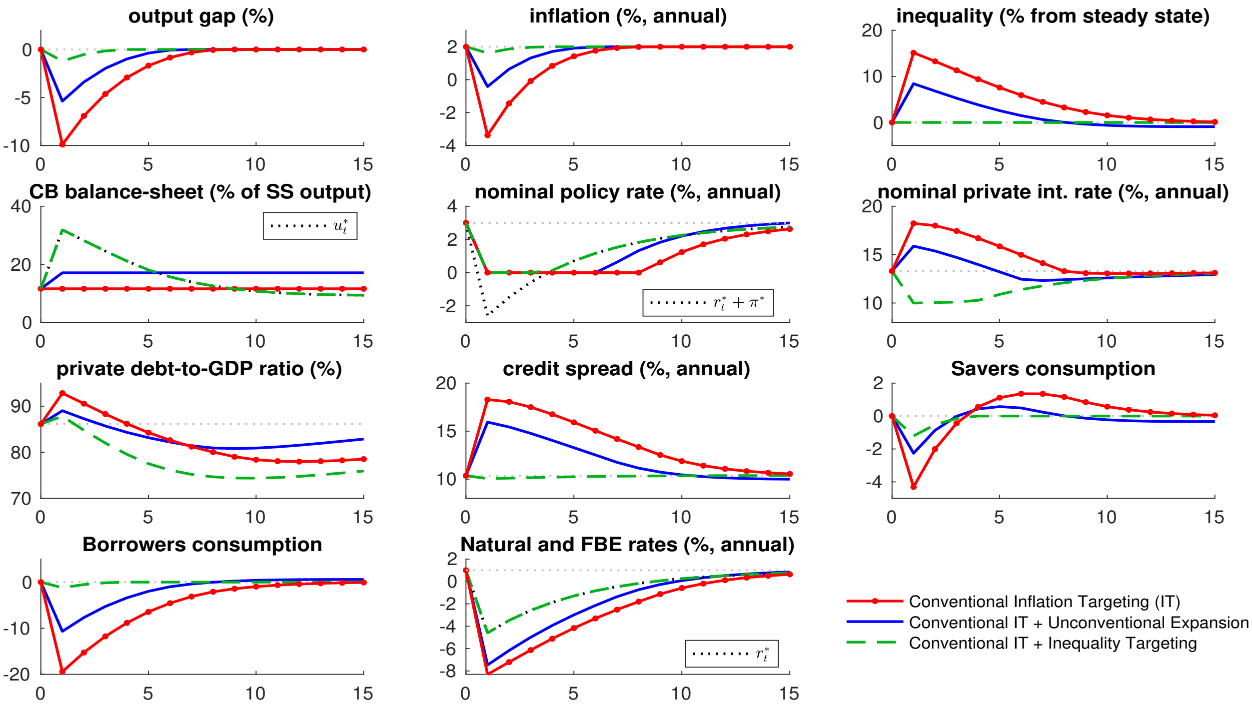

Figure 4: The response of the economy to a natural interest rate fall consistent with the Great Recession. Blue solid line: conventional inflation targeting plus unconventional balance-sheet expansion; red line with dots: conventional inflation targeting; green dashed line: conventional inflation targeting plus unconventional inequality targeting.

sumption falls between two and three times more. This occurs because the decline in borrowers' consumption is deeper and more persistent in the absence of an unconventional response, thereby increasing the consumption risk faced by savers. As a result, savers increase their precautionary savings and reduce their spending. This effect dominates the intertemporal substitution induced by the policy rate and accounts for about 25% of the short-run decline in aggregate output.

The case of "unconventional inequality targeting" clarifies that, by stabilizing consumption inequality, this policy regime effectively shuts down the idiosyncratic-risk channel of transmission of the shock, which would otherwise amplify the recessionary and deflationary consequences of the crisis. The result is a substantially more stable output gap and inflation, and a faster private deleveraging. This latter outcome is worth emphasizing. Intuitively, it is due to the interaction between a persistently lower interest rate on borrowing (which reduces the pressures on rollovers) with the shutdown of the idiosyncratic risk channel (which would otherwise stimulate additional borrowing for anticipative motives). Moreover, it is particularly interesting because it highlights that a private debt deleveraging cycle needs not be associated with large recessions, as long as unconventional policy responds appropriately.

This observation challenges one of the main messages of earlier contributions like Benigno et al (2020), that in a deleveraging episode the policy response should be more aggressive and the ZIRP longer than in the standard RANK model because the endogenous response of the natural interest rate makes it more persistent. The discussion above suggests that this needs not be the case. If the central bank deploys its unconventional tools—whether as in our baseline policy scenario or in the

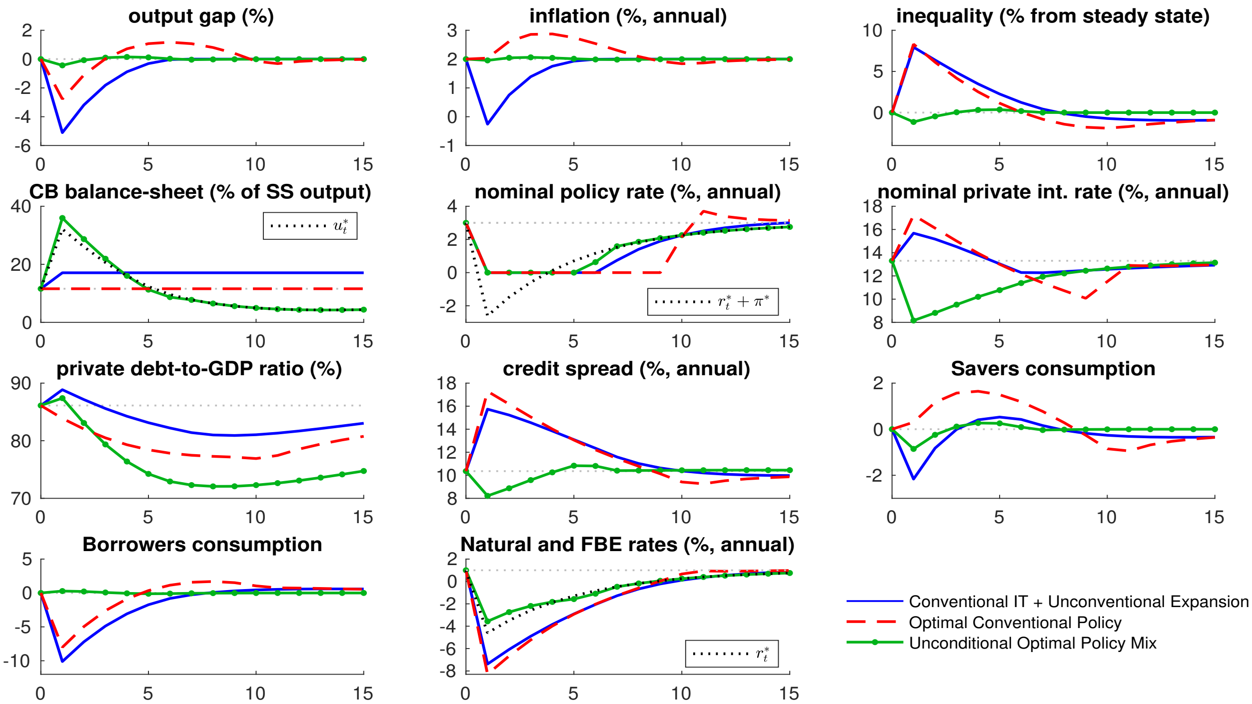

Figure 5: The response of the economy to a natural interest rate fall consistent with the Great Recession. Blue solid line: conventional inflation targeting plus unconventional balance-sheet expansion; red dashed line: optimal conventional policy; green line with dots: unconditional optimal conventional-unconventional policy mix.

inequality-targeting regime—the endogeneity of the natural interest rate actually makes its decline milder and less persistent: unconventional policy can grant the additional aggressiveness needed to respond to the shock without the need for a longer zero-interest rate policy. In fact, it actually makes its duration shorter—and only related to aggregate shocks in the case of inequality targeting.

To explore whether there is normative value in this implication, Figure 5 displays the response of the economy in our baseline policy scenario (solid blue line), and compares it to the optimal policy under full commitment conditional on using the conventional tool only (red dashed line) and the unconditional one, i.e. when also using the unconventional tool (green line with dots).[38].

As in Benigno et al (2020), the optimal policy when only using the conventional tool includes a commitment to keep the policy rate at the ZLB for longer than in the standard RANK model, resulting in an earlier output boom and an inflation rate that is above target throughout the liquidity trap, supported by a less persistent fall in the endogenous natural rate of interest.

Such commitment improves also on our baseline policy scenario that includes an unconventional expansion, essentially through a substantially lower interest rate on borrowing in the last quarters of the liquidity trap. As a result, the consumption of borrowers declines by less and for a shorter period of time, and it experiences a boom before exiting the ZIRP. Note the role of idiosyncratic uncertainty: the more favorable outlook for borrowers reduces consumption risk for the savers, who increase consumption as the intertemporal substitution induced by the interest-rate policy dominates the precautionary-saving motive induced by the idiosyncratic-risk channel of transmission

[38]For details on the optimal policy problem under commitment in the general model, please refer to Appendix D.

of the shock. This is also reflected in the dynamics of consumption inequality, that at first rises as much as in our baseline scenario but then it falls below steady state earlier and by a larger extent.

The fact that consumption inequality rises on impact, however, suggests that the "optimal conventional policy" may not be *unconditionally* optimal. Indeed, this regime is constrained by the assumption that the central bank's balance sheet does not respond. Relaxing this assumption and computing the "unconditional optimal policy", reveals the key role of the unconventional dimension. The commitment to keep the policy rate at zero is now much weaker, and complemented by a strong expansion of the central bank's balance sheet, implying $\hat{u}_t > u_t^*$ for as long as the nominal natural rate is negative. The result of this optimal policy mix, where the *forward guidance* on the policy rate is mostly replaced by an unconventional balance-sheet expansion, is a substantial stabilization of all welfare-relevant variables compared to the optimal conventional policy. The optimal policy mix has several elements in common with the unconventional "inequality targeting regime": it uses the balance-sheet expansion to improve the outlook of borrowers through a reduction in the borrowing interest rate that allows for a faster and deeper deleveraging process and at the same time for a milder and less persistent decline in the natural interest rate. But it does so to a larger extent, compared to the "inequality targeting regime": the discount-factor component of the fall in natural rate requires indeed to induce a stronger decline in the credit spread that trades off some inequality stability for a more stable output gap and inflation, through the idiosyncratic-risk channel.

An interesting corollary of this result is that introducing unconventional policy and idiosyncratic risk reduces the relative attractiveness of inflation policy compared to the RANK model, advocated by Benigno et al (2020) on the grounds that it benefits borrowers. In our economy, it is unconventional policy that should take care of borrowers, while interest-rate policy should focus on shocks with primarily aggregate effects—such as discount-factor shocks.

5 Conclusion

This paper studies the monetary-policy implications of an economy where households are heterogeneous and face idiosyncratic risk, financial intermediaries are limited by some leverage constraints, and the central bank controls both the interest rate on its reserves and the size of its balance sheet.

Accounting for idiosyncratic risk and cyclical inequality opens room for two additional channels of transmission of central banks' balance-sheet policies, related to consumption risk. The idiosyncratic-risk channel in particular is key for the transmission of persistent balance-sheet policies: improving the outlook for borrowers reduces consumption risk for savers, which respond cutting their precautionary savings and expanding their current spending. This can critically amplify the expansionary effect of unconventional monetary policy, that initially only affects the borrowers.

Through this channel, unconventional monetary policy improves the ability of the central bank to anchor private-sector expectations and rule out endogenous instability. Appropriately specified balance-sheet policy rules allow the central bank to implement a (locally) unique rational-expectations equilibrium even in the case of an interest-rate peg, or a permanent liquidity trap.

Unconventional monetary policy allows the central bank to achieve prices stability even in the face of shocks that conventional policy would find impossible to sterilize due to the existence of an effective lower bound on the policy interest rate. The unconditional optimal policy includes an unconventional balance-sheet component that improves the ability of the central bank to reduce fluctuations in inflation and the output gap during a liquidity trap. Importantly, and differently from recent related literature, it promotes a shorter rather than longer optimal duration of zero interest-rate policies, reducing also the appeal of front-loaded inflation policy to improve the welfare of borrowers.

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Appendix

A The Complete Model

We consider a closed-economy model including two types of households—a mass z of borrowers and a mass $1-z$ of savers—a financial intermediation sector, a continuum of monopolistic firms, a fiscal authority and a central bank.

A.1 Financial Intermediaries

Financial intermediaries collect from the savers the stock of one-period nominal deposits D, and the stock of nominal net worth N. They allocate these resources in a portfolio of nominal assets including central-bank reserves R^f and one-period bonds issued by private borrowers, $B^f = \int_z B^f(j)dj$, where individual borrowers are indexed by j. Let $x \equiv \frac{X}{P}$ for $x = b$, d, n and $u \equiv \frac{R}{P}$. The balance sheet of the banking sector in period t, in real terms, therefore reads

$$b_t^f + u_t^f = d_t + n_t. \tag{A.1}$$

In lending to the private sector, intermediaries face the following leverage constraint:

$$b_t^f \leq \theta_t n_t. \tag{A.2}$$

with θ_t following an exogenous stochastic process.

As in Benigno et al. (2020), the banks remunerate nominal deposits and net worth at the rate i^D, which they take as given, but they also face an additional cost of raising net worth above some threshold $\bar{n}$ (which we assume equal to its steady-state level). To allow for a potential role of individual borrowing on the equilibrium credit spread, we specify the cost of equity funding as

$$cef_t \equiv \theta_t f\left(\frac{n_t}{\bar{n}}\right) q_t,$$

with

$$q_t \equiv z^{-1} \int_z q\left(\frac{b_t^f(j)}{\bar{b}_t^f}\right) dj.$$

The function $f(x)$—with $f(x) = 0$ for $x \leq 1$ and $f'(x)$, $f''(x) > 0$ for $x \geq 1$—captures the convex costs of raising aggregate capital in excess of the threshold, as in Benigno et al (2020). The additional function $q(y)$—with $q(y) = 1$ for $y \leq 1$, $q'(y)$, $q''(y) > 0$ for $y \geq 1$—captures the idea that for banks is more costly to lend a single borrower more than the average level consistent with the threshold capital $\bar{n}$, given the current leverage ratio θ_t: i.e. $\bar{b}_t^f \equiv \bar{n}\theta_t/z$.[39]

On the other side of the balance sheet, they collect the gross return on the private assets, i.e.

[39] Note that we relax the restriction that $f'(x)$, $q'(y) = 0$ for $x = y = 1$—assumed in Benigno et al (2020) to support a zero steady-state spread—and use the additional flexibility to calibrate an empirically sensible steady-state spread.

$1+i^B(j)$, for each $j \in [0, z]$, and on central bank reserves, i.e. $1+i^R \geq 1$, both of which they take as given. As a consequence, the intermediary's real profits matured in period t are

$$\frac{\Pi_{t+1}^f}{P_t} \equiv \int_z \left(1+i_t^B(j)\right) b_t^f(j)dj + (1+i_t^R)u_t^f - (1+i_t^D)d_t - (1+i_t^D)n_t \left[1+\theta_t f\left(\frac{n_t}{\bar{n}}\right) q_t\right]. \quad \text{(A.3)}$$

The objective function of financial intermediaries in period t is the expected discounted level of profits they will remit to savers in period $t+1$:

$$E_t\left\{\Lambda_{t,t+1}^s \frac{\Pi_{t+1}^f}{P_t}\right\} = \left[\int_z \frac{1+i_t^B(j)}{1+i_t^D} b_t^f(j)dj - b_t^f\right] + \left[\frac{1+i_t^R}{1+i_t^D} - 1\right] u_t^f - n_t\theta_t f\left(\frac{n_t}{\bar{n}}\right) q_t, \quad \text{(A.4)}$$

where we used the balance-sheet constraint (A.1) and $E_t\left\{\Lambda_{t,t+1}^s\right\} = (1+i_t^D)^{-1}$ from the optimality conditions of the savers. Banks then seek to maximize (A.4) subject to the leverage constraint (A.2).

In the main text we study two cases. In the special case of Assumption 1, where households transfer to banks a constant amount of real capital (i.e. $n_t = \bar{n}$ for all t), the banks choose the composition and size of their asset portfolio (i.e. b_t^f and u_t^f) and their objective simplifies to

$$E_t\left\{\Lambda_{t,t+1}^s \frac{\Pi_{t+1}^f}{P_t}\right\} = \left[\frac{1+i_t^B}{1+i_t^D} - 1\right] b_t^f + \left[\frac{1+i_t^R}{1+i_t^D} - 1\right] u_t^f, \quad \text{(A.5)}$$

where we anticipate the result that, since $f(1) = 0$, there is no role for individual debt levels in the equilibrium interest rate on private assets: $i^B(j) = i^B$ and $b^f(j) = b^f/z$. The solution to this problem then implies that the interest rate on deposits equals at all times the interest rate on central-bank reserves, $i_t^D = i_t^R \geq 0$, while private bonds pay a premium in case the leverage constraint binds:

$$\frac{1+i_t^B}{1+i_t^R} = 1+\zeta_t \geq 1, \quad \text{(A.6)}$$

where ζ_t is the Lagrange multiplier associated to constraint (A.2).

In the general case of Assumption 3, instead, the solution to the banks' problem is modified as follows. The equilibrium spread for borrower $j \in [0, z]$ is affected by their individual debt level:

$$\frac{1+i_t^B(j)}{1+i_t^R} = 1+\zeta_t + \frac{n_t}{\bar{n}} f\left(\frac{n_t}{\bar{n}}\right) q'\left(\frac{b_t^f(j)}{\bar{b}_t^f}\right), \quad \text{(A.7)}$$

and the solution now includes an additional first-order condition, with respect to banks' capital:

$$q_t\left[f\left(\frac{n_t}{\bar{n}}\right) + \frac{n_t}{\bar{n}} f'\left(\frac{n_t}{\bar{n}}\right)\right] = \zeta_t. \quad \text{(A.8)}$$

Combining (A.8) with (A.7), (A.1) and (A.2) holding with equality implies

$$\frac{1+i_t^B(j)}{1+i_t^R} = 1+F\left(\frac{n_t}{\bar{n}}, \frac{b_t^f(j)}{\bar{b}_t^f}\right) = 1+F\left(\frac{b_t - u_t}{\bar{n}\theta_t}, \frac{b_t^f(j)}{\bar{b}_t^f}\right), \quad \text{(A.9)}$$

where $F(x, y) \equiv [f(x) + xf'(x)]q(y) + xf(x)q'(y)$, and where in the second equality we used (A.2) holding with equality and the balance-sheet constraint of the central bank, implying $b_t = b_t^f + u_t$.

Finally, using on (A.9) the result that, in equilibrium, all borrowers are identical, which implies $b_t^f(j)/\bar{b}_t^f = b_t^f/z\bar{b}_t^f = n_t/\bar{n}$, yields

$$\frac{1+i_t^B}{1+i_t^R} = 1 + \mathcal{S}\left(\frac{b_t - u_t}{\bar{n}\theta_t}\right), \tag{A.10}$$

with $\mathcal{S}\left(\frac{b_t-u_t}{\bar{n}\theta_t}\right) \equiv F\left(\frac{n_t}{\bar{n}}, \frac{n_t}{\bar{n}}\right) = F\left(\frac{b_t-u_t}{\bar{n}\theta_t}, \frac{b_t-u_t}{\bar{n}\theta_t}\right)$.

A.2 Households

Households belong to either one of two types: savers (in mass $1-z$, denoted with an index "s") and borrowers (in mass z, denoted with an index "b"). Each saver faces a probability $1-p_s$ of becoming a borrower as the next period begins, and each borrower a probability $1-p_b$ of becoming a saver. To keep the relative mass of the two agent types constant over time, we impose the restriction $(1-z)(1-p_s) = z(1-p_b)$. Savers and borrowers share the same period-utility function, $u_k \equiv \xi[U(C_k(j)) - V(L_k(j))]$, with $k = s$, b and where ξ is an intertemporal disturbance. A key difference between savers and borrowers come from their average labor income: each household j belonging to type $k = s, b$ is endowed with a stochastic idiosyncratic labor-market status $\varepsilon_{k,t}(j) = \{0, 1\}$, whereby their time-$t$ labor income is

$$\mathcal{I}_{k,t}^L(j) \equiv \varepsilon_{k,t}(j) W_{k,t} L_{k,t}(j),$$

with $j \in [0, z]$ if $k = b$ and $j \in (z, 1]$ if $k = s$. The idiosyncratic labor-income uncertainty is i.i.d. within each type, with

$$\text{prob}\left(\varepsilon_{k,t}(j) = 1 | k = b\right) = \varpi_t < \text{prob}\left(\varepsilon_{k,t}(j) = 1 | k = s\right) = 1,$$

where the probability of the high-income state for the borrowers is pro-cyclical: $\varpi_t = g(x_t)$, with $g(\cdot) \in [0, 1]$, $g'(\cdot) > 0$ and $g(0) \equiv \varpi \in [0, 1]$ denoting the steady-state probability of the good employment status, and $x_t \equiv \log(Y_t/Y_t^*)$ the gap between real (Y) and potential output (Y^*).

To facilitate aggregation of the supply side of the economy in the non-linear equilibrium and in a linear approximation around an unequal steady state, we assume that the period-utility function is exponential in consumption C and isoelastic in hours worked L, as in Benigno and Nisticò (2017) and Benigno et al (2020), among others:[40]

$$U(C_{k,t}) \equiv 1 - \exp(-\sigma C_{k,t}) \qquad\qquad V(L_{k,t}) \equiv \frac{L_{k,t}^{1+\varphi}}{1+\varphi} \tag{A.11}$$

for $k = s$, b, and some positive parameter σ, and where consumption is the usual Dixit-Stiglitz

[40] See also Acharya and Dogra (2020) in the context of heterogeneous-agents models. Standard boundary conditions on exogenous shocks rule out the possibility of negative consumption levels, starting from a well defined steady state.

bundle

$$C \equiv \left[\int_0^1 C(i)^{\frac{1}{1+\mu}} di\right]^{1+\mu}, \tag{A.12}$$

with $C(i)$ denoting the consumption of the differentiated good of brand i, $\mu > 0$ the net price markup and $(1+\mu)/\mu > 1$ the elasticity of substitution between any two brands in the continuum indexed by $i \in [0, 1]$. As in Bilbiie (2018), a utilitarian family head maximizes the average lifetime utility across all agents

$$\mathcal{U}_t = (1-z)u_{s,t} + zu_{b,t} + \beta E_t \mathcal{U}_{t+1} \tag{A.13}$$

subject to a limited risk-sharing constraint that allows all agents within each type—but not across types—to pool their resources and obligations and share the same level of consumption.[41]

A.2.1 Savers

Savers are subject to the following flow-budget constraint

$$P_t C_{s,t} + (1-p_s)(1+i_t^B)B_{t-1}^h + D_t^h + N_t^h = W_{s,t}L_{s,t} + p_s(1+i_{t-1}^D)(D_{t-1}^h + N_{t-1}^h) + \Pi_t - T_{s,t}. \tag{A.14}$$

The nominal resources available to savers at the beginning of each period t therefore include the labor income $W_{s,t}L_{s,t}$, the payoff from the portfolio of deposits and bank's capital from the previous period $(1+i_{t-1}^D)(D_{t-1}^h + N_{t-1}^h)$ held by the share p_s of savers that did not turn borrowers, the per-capita nominal profits $\Pi_t \equiv (1-z)^{-1}(\Pi_t^p + \Pi_t^f)$ remitted by the monopolistic producers and the financial intermediaries—both owned by the savers—net of taxes/transfers $T_{s,t}$, with P_t the general consumption price level. The savers use these resources to purchase a bundle of consumption goods $C_{s,t}$, save in one-period nominal deposits D_t^h, transfer nominal equity N_t^h to financial intermediaries, and share *pro quota* the burden of paying off the debt $\int_z B_{t-1}^h(j)dj$ brought by the mass $z(1-p_b)$ of borrowers that have turned savers at the beginning of period t, where we used the restriction $(1-z)(1-p_s) = z(1-p_b)$ and anticipated the result that borrowers are all identical within their set, i.e. $B^h(j) = B^h$.

The optimal choice of consumption, hours worked, deposits and banks capital implies the Euler equation

$$\xi_t U_c(C_{s,t}) = \beta E_t \left\{ \frac{1+i_t^D}{1+\pi_{t+1}} \xi_{t+1} \left[p_s U_c(C_{s,t+1}) + (1-p_s)U_c(C_{b,t+1}) \right] \right\} \tag{A.15}$$

where π_{t+1} is the net inflation rate between period t and $t+1$, and the labor supply

$$\frac{V_l(L_{s,t})}{U_c(C_{s,t})} = \frac{W_{s,t}}{P_t}. \tag{A.16}$$

Moreover, the interest-rate on short-term deposits satisfies the following no-arbitrage condition

$$1 = (1+i_t^D)E_t\left\{\Lambda_{t,t+1}^s\right\}, \tag{A.17}$$

[41] Analogous implications would follow from an imperfect-insurance scheme as, e.g., in Cúrdia and Woodford (2016).

where $\Lambda^s_{t,t+1}$ denotes the nominal stochastic discount factor used by savers, defined as

$$\Lambda^s_{t,t+1} \equiv \beta \frac{\xi_{t+1}}{\xi_t} \frac{p_s U_c(C_{s,t+1}) + (1-p_s) U_c(C_{b,t+1})}{(1+\pi_{t+1}) U_c(C_{s,t})}. \tag{A.18}$$

A.2.2 Borrowers

Borrowers are subject to the following flow-budget constraint

$$P_t C_{b,t} + p_b(1+i^B_t) B^h_{t-1} = \varpi_t W_{b,t} L_{b,t} + (1-p_b)(1+i^D_{t-1})(D^h_{t-1} + N^h_{t-1}) + B^h_t + T_{b,t}. \tag{A.19}$$

The nominal resources available to borrowers at the beginning of each period t therefore include the average labor income $\varpi_t W_{b,t} L_{b,t}$, the per-capita share of payoff on the portfolio of deposits and bank capital from the previous period $(1+i^D_{t-1})(D^h_{t-1} + N^h_{t-1})$ brought by the mass $(1-z)(1-p_s)$ of savers that have turned borrowers at the beginning of the period—where we used the restriction $(1-z)(1-p_s) = z(1-p_b)$—the resources borrowed selling private debt B^h_t and the transfers $T_{b,t}$ received by the fiscal authority. The borrowers use these resources to purchase a bundle of consumption goods $C_{b,t}$, and pay off the debt B^h_{t-1} accumulated in the previous period by the share p_b of borrowers that have not turned savers at the beginning of period t.

As each individual borrower j may be charged a different rate depending on their specific debt level, their choice of debt internalizes this effect by considering the equilibrium spread (A.9) as an additional constraint to optimization. The first-order conditions with respect to consumption and individual debt then imply the Euler equation

$$\xi_t U_c(C_{b,t}(j)) = \beta E_t \left\{ \Big(1 + \epsilon_t(j)\Big) \frac{1+i^B_t(j)}{1+\pi_{t+1}} \xi_{t+1} \Big[p_b U_c(C_{b,t+1}(j)) + (1-p_b) U_c(C_{s,t+1}) \Big] \right\}, \tag{A.20}$$

where $i^B_t(j)$ is the borrowing rate for the individual j, and

$$\epsilon_t(j) \equiv \frac{b^h_t(j)}{\bar{b}^f_t} \frac{F_2\left(\frac{b_t - u_t}{\bar{n}\theta_t}, \frac{b^f_t(j)}{\bar{b}^f_t}\right)}{1 + F\left(\frac{b_t - u_t}{\bar{n}\theta_t}, \frac{b^f_t(j)}{\bar{b}^f_t}\right)}$$

its elasticity to individual debt, as in Benigno et al (2020), with $F_2(\cdot,\cdot)$ denoting the partial derivative of function $F(\cdot,\cdot)$ with respect to its second argument. Perfect risk-sharing within the set borrowers implies in equilibrium $i^B_t(j) = i^B_t$, $\epsilon_t(j) = \epsilon_t$ and $C_{b,t}(j) = C_{b,t}$, for each t, and thus:

$$1 = (1+\epsilon_t)(1+i^B_t) E_t \left\{ \Lambda^b_{t,t+1} \right\}, \tag{A.21}$$

where $\Lambda^b_{t,t+1}$ denotes the nominal stochastic discount factor used by borrowers, defined as

$$\Lambda^b_{t,t+1} \equiv \beta \frac{\xi_{t+1}}{\xi_t} \frac{p_b U_c(C_{b,t+1}) + (1-p_b) U_c(C_{s,t+1})}{(1+\pi_{t+1}) U_c(C_{b,t})}. \tag{A.22}$$

The optimal choice of consumption and the labor supply finally implies

$$\frac{V_l(L_{b,t})}{U_c(C_{b,t})} = \varpi_t \frac{W_{b,t}}{P_t}. \tag{A.23}$$

A.3 Firms

A continuum of firms of measure one produces each one brand of differentiated goods using the linear technology

$$Y_t(i) = A_t L_t(i) \tag{A.24}$$

for all brands $i \in [0, 1]$. The labor input combines the hours worked of savers and borrowers through the Cobb-Douglas technology

$$L_t(i) = [L_{s,t}(i)]^{1-z} [L_{b,t}(i)]^z, \tag{A.25}$$

which implies that the wage bills for each type of labor is the same as the average wage bill, $W_{s,t}L_{s,t} = W_{b,t}L_{b,t} = W_t L_t$ where $W_t = W_{s,t}^{1-z} W_{b,t}^z$.

Firms set their price according to the Calvo mechanism, whereby each period a share $\alpha \in [0, 1]$ of firms passively index their price to the inflation target π^* while the remaining share $1-\alpha$ optimally sets the price at level P_t^*. Given this structure, the equilibrium inflation rate then satisfies

$$1 = (1-\alpha)\left(\frac{P_t}{P_t^*}\right)^{1/\mu} + \alpha\left(\frac{1+\pi_t}{1+\pi^*}\right)^{1/\mu}. \tag{A.26}$$

A common optimal price level P_t^* is chosen by all firms that are able to reset their price at t, as it maximizes the expected discounted stream of future profits

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty} \alpha^{t-t_0} \Lambda_{t_0,t}^s Y_t(i)\left[(1+\pi^*)^{t-t_0}\frac{P_t(i)}{P_t} - MC_t\right]\right\}, \tag{A.27}$$

subject to the demand for brand i, $Y_t(i) = (P_t(i)/P_t)^{-(1+\mu)/\mu} Y_t$, in which aggregate output satisfies the resource constraint

$$Y_t = (1-z)C_{s,t} + zC_{b,t}. \tag{A.28}$$

In the objective of the firm (A.27), the stochastic discount factor used is that of savers, which own the firms, and real marginal costs are given by

$$MC_t = (1-\tau)\frac{W_t}{P_t A_t}, \tag{A.29}$$

where τ is an employment subsidy.

The solution to the firms' problem implies, also using (A.26):

$$\left(\frac{1-\alpha\left(\frac{1+\pi_t}{1+\pi^*}\right)^{1/\mu}}{1-\alpha}\right)^{-\mu} = (1+\mu)\frac{F_t}{K_t}, \tag{A.30}$$

with

$$F_t \equiv E_t\left\{\sum_{T=t}^{\infty}\alpha^{T-t}\Lambda^s_{t,T}Y_T\left(\frac{P_T}{P_t\left(1+\pi^*\right)^{T-t}}\right)^{\frac{1+\mu}{\mu}}MC_t\right\}$$
$$= Y_tMC_t + \alpha E_t\left\{\Lambda^s_{t,t+1}\left(\frac{1+\pi_{t+1}}{1+\pi^*}\right)^{\frac{1+\mu}{\mu}}F_{t+1}\right\} \tag{A.31}$$

$$K_t \equiv E_t\left\{\sum_{T=t}^{\infty}\alpha^{T-t}\Lambda^s_{t,T}Y_T\left(\frac{P_T}{P_t\left(1+\pi^*\right)^{T-t}}\right)^{-1/\mu}\right\}$$
$$= Y_t + \alpha E_t\left\{\Lambda^s_{t,t+1}\left(\frac{1+\pi_{t+1}}{1+\pi^*}\right)^{-1/\mu}K_{t+1}\right\}. \tag{A.32}$$

In equilibrium, firms' real marginal costs follow from aggregation of the labor supply equations of savers and borrowers, which the specification of preferences (A.11) and technology (A.25) keep tractable:

$$MC_t = (1-\tau)\frac{W_t}{P_tA_t} \tag{A.33}$$
$$\frac{W_t}{P_t} = \frac{\varpi_t^{-z}(Y_t\Delta^p_t)^{\varphi}}{\sigma\exp(-\sigma Y_t)A_t^{\varphi}}, \tag{A.34}$$

where we have also used the production function (A.24), the aggregator (A.12) and the resource constraint (A.28), and where Δ^p_t is an index of relative-price dispersion across firms

$$\Delta^p_t = \int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\mu}{\mu}}di, \tag{A.35}$$

which evolves according to

$$\Delta^p_t = \alpha\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\frac{1+\mu}{\mu}}\Delta^p_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{1+\pi_t}{1+\pi^*}\right)^{1/\mu}}{1-\alpha}\right)^{1+\mu}. \tag{A.36}$$

A.4 The Fiscal Authority

The government consists of a fiscal authority, running a balanced budget every period, and a central bank in charge of monetary policy. The fiscal authority provides transfers and charges taxes. The (nominal) transfers include the employment subsidy to firms $\tau W_t L_t$ and a lump-sum transfer to borrowers $T_{b,t}$ that partially limits their liability position and simplifies their equilibrium.[42] These transfers are financed using lump-sum taxes on the savers and the remittances T_t^c received by the central bank. The budget constraint of the fiscal authority, in nominal terms, is therefore:

$$\tau W_t L_t + z T_{b,t} = T_t^c + (1-z) T_{s,t}. \tag{A.37}$$

A.5 Aggregate Equilibrium

Equilibrium in the asset markets requires, for each period t implies

$$(1-z) D^h = D \tag{A.38}$$

$$(1-z) N^h = N \tag{A.39}$$

$$z B^h = B^f + B^c = B \tag{A.40}$$

$$R = R^f. \tag{A.41}$$

Define $u_t \equiv \frac{R_t}{P_t}$, $b_t \equiv \frac{B_t}{P_t}$, $b_t^f \equiv \frac{B_t^f}{P_t}$, $b_t^c \equiv \frac{B_t^c}{P_t}$, $d_t \equiv \frac{D_t}{P_t}$, $d_t^h \equiv \frac{D_t^h}{P_t}$, $n_t \equiv \frac{N_t}{P_t}$, $n_t^h \equiv \frac{N_t^h}{P_t}$, and $w_t \equiv \frac{W_t}{P_t}$. Under Assumptions 1 and 2, an equilibrium is a collection of seventeen stochastic processes

$$\{Y_t,\, \pi_t,\, C_{s,t},\, C_{b,t},\, i_t^D,\, i_t^B,\, i_t^R,\, b_t,\, u_t,\, \Lambda_{t,t+1}^s,\, \zeta_t,\, MC_t,\, \Delta_t^p,\, F_t,\, K_t,\, w_t,\, L_t\}_{t=t_0}^{\infty}$$

that satisfy the following fifteen restrictions, expressed in real terms

$$\bar{n}\theta_t + u_t = b_t \tag{A.42}$$

$$1 + i_t^B = (1 + i_t^R)(1 + \zeta_t) \tag{A.43}$$

$$C_{b,t} = z^{-1} b_t + \left(\varpi_t - \frac{\nu}{z}\right) w_t L_t + \frac{\nu}{z} Y_t - \bar{T} \tag{A.44}$$

$$\xi_t U_c(C_{b,t}) = \beta E_t \left\{ \frac{1 + i_t^B}{1 + \pi_{t+1}} \xi_{t+1} \left[p_b U_c(C_{b,t+1}) + (1 - p_b) U_c(C_{s,t+1}) \right] \right\} \tag{A.45}$$

$$\xi_t U_c(C_{s,t}) = \beta E_t \left\{ \frac{1 + i_t^D}{1 + \pi_{t+1}} \xi_{t+1} \left[p_s U_c(C_{s,t+1}) + (1 - p_s) U_c(C_{b,t+1}) \right] \right\} \tag{A.46}$$

$$1 + i_t^D = 1 + i_t^R \geq 1 \tag{A.47}$$

$$Y_t = (1-z) C_{s,t} + z C_{b,t} \tag{A.48}$$

[42] A similar scheme is assumed in Sims et al. (2023).

$$MC_t = (1-\tau)\frac{\varpi_t^{-z}(Y_t\Delta_t^p)^{\varphi}}{v\exp(-vY_t)A_t^{1+\varphi}} \tag{A.49}$$

$$\left(\frac{1-\alpha\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\epsilon-1}}{1-\alpha}\right)^{\frac{1}{1-\epsilon}} = \frac{\epsilon}{\epsilon-1}\frac{F_t}{K_t}, \tag{A.50}$$

$$F_t = Y_t MC_t + \alpha E_t\left\{\Lambda^s_{t,t+1}\left(\frac{1+\pi_{t+1}}{1+\pi^*}\right)^{\epsilon} F_{t+1}\right\} \tag{A.51}$$

$$K_t = Y_t + \alpha E_t\left\{\Lambda^s_{t,t+1}\left(\frac{1+\pi_{t+1}}{1+\pi^*}\right)^{1-\epsilon} K_{t+1}\right\} \tag{A.52}$$

$$w_t = \frac{\varpi_t^{-z}(Y_t\Delta_t^p)^{\varphi}}{v\exp(-vY_t)A_t^{\varphi}} \tag{A.53}$$

$$\Delta_t^p = \alpha\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\epsilon}\Delta_{t-1}^p + (1-\alpha)\left(\frac{1-\alpha\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\epsilon-1}}{1-\alpha}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{A.54}$$

$$Y_t\Delta_t^p = A_t L_t \tag{A.55}$$

$$\Lambda^s_{t,t+1} \equiv \frac{p_s U_c(C_{s,t+1}) + (1-p_s)U_c(C_{b,t+1})}{(1+\pi_{t+1})U_c(C_{s,t})}, \tag{A.56}$$

for a given vector of exogenous processes $\{\mathbf{X}_t\}_{t=t_0}^{\infty}$ with $\mathbf{X}_t \equiv (\xi_t,\, A_t,\, \theta_t)$, and where we focus on equilibria where the banks' leverage constraint is always binding, implying $b_t^f = \bar{n}\theta_t$. With fifteen restrictions to determine seventeen processes, we have two degrees of freedom that we can exploit to specify the two dimensions of monetary policy.

Under the general specification of Assumptions 3 and 4, in turn, the system of relevant equilibrium restriction includes equations (A.46) through (A.56), plus the following:

$$n_t\theta_t + u_t = b_t \tag{A.57}$$

$$\frac{1+i_t^B}{1+i_t^R} = 1 + \mathcal{S}\left(\frac{b_t - u_t}{\bar{n}\theta_t}\right) \tag{A.58}$$

$$b_t = \left[p_b\frac{1+i_{t-1}^B}{1+i_{t-1}^R} - (1-p_s)\right]\frac{1+i_{t-1}^R}{1+\pi_t}b_{t-1} + zC_{b,t} - (z\varpi_t - \nu)\,w_tL_t - \nu Y_t + \bar{T} \tag{A.59}$$

$$\xi_t U_c(C_{b,t}) = \beta E_t\left\{(1+\epsilon_t)\frac{1+i_t^B}{1+\pi_{t+1}}\xi_{t+1}\Big[p_bU_c(C_{b,t+1}) + (1-p_b)U_c(C_{s,t+1})\Big]\right\} \tag{A.60}$$

$$\epsilon_t(j) \equiv \frac{b_t}{\bar{n}\theta_t}\frac{F_2\left(\frac{b_t-u_t}{\bar{n}\theta_t},\frac{b_t-u_t}{\bar{n}\theta_t}\right)}{1+F\left(\frac{b_t-u_t}{\bar{n}\theta_t},\frac{b_t-u_t}{\bar{n}\theta_t}\right)}. \tag{A.61}$$

Once we specify the two dimensions of monetary policy, the above system of sixteen restrictions can determine the equilibrium set of eighteen processes

$$\{Y_t,\, \pi_t,\, C_{s,t},\, C_{b,t},\, i_t^D,\, i_t^B,\, i_t^R,\, b_t,\, u_t,\, \Lambda_{t,t+1}^s,\, \epsilon_t,\, MC_t,\, \Delta_t^p,\, F_t,\, K_t,\, w_t,\, L_t,\, n_t\}_{t=t_0}^{\infty}$$

for a given vector of exogenous processes $\{\mathbf{X}_t\}_{t=t_0}^{\infty}$ with $\mathbf{X}_t \equiv (\xi_t,\, A_t,\, \theta_t)$, and where again we focus on equilibria where the banks' leverage constraint is always binding.

A.6 The Steady State

We assume a level of employment subsidies such that the long-run monopolistic distortions are offset: $\tau^* = \mu/(1+\mu)$. Under this employment subsidy, the long-run level of output satisfies

$$\frac{\varpi^{-z}\bar{Y}^{\varphi}}{\sigma \exp(-\sigma \bar{Y})\bar{A}^{1+\varphi}} = 1. \tag{A.62}$$

To simplify notation, we normalise the steady-state level of the productivity index $\bar{A}$ such that, from equation (A.62), we have $\bar{Y} = 1$. Equation (A.62), together with the steady-state aggregate labor supply and production function, also implies $\bar{w}\bar{L} = 1$. Fiscal redistribution in the steady state implies, for both the simple and the general model, the following distribution of consumption:

$$\bar{C}_b = \varpi - z^{-1}\bar{b}\left(\frac{\bar{\imath}^B - \pi^*}{1+\pi^*}\right) \tag{A.63}$$

$$\bar{C}_s = \frac{1 - z\bar{C}_b}{1-z}. \tag{A.64}$$

The steady-state versions of the Euler equations (A.46) and (A.60) then imply

$$1 + \pi^* = \beta\Gamma_s(1 + \bar{\imath}^R) \tag{A.65}$$

$$1 + \pi^* = \beta\Gamma_b(1 + \bar{\imath}^B)(1 + \bar{\epsilon}) \tag{A.66}$$

where we have used $i^D = i^R \geq 0$ and we defined

$$\Gamma_s \equiv p_s + (1 - p_s)\exp\left[(\Gamma - 1)\,\sigma_b\right] \tag{A.67}$$

$$\Gamma_b \equiv p_b + (1 - p_b)\exp\left[(1 - \Gamma)\,\sigma_b\right], \tag{A.68}$$

with $\sigma_b \equiv \sigma\bar{C}_b$. Accordingly, we can write

$$\frac{1 + \bar{\imath}^B}{1 + \bar{\imath}^R} = (1 + \bar{\epsilon})^{-1}\frac{\Gamma_s}{\Gamma_b} \geq 1, \tag{A.69}$$

where the inequality holds if the value of Γ implies $\Gamma_s \geq \Gamma_b$ and $\bar{\epsilon}$ is sufficiently small.

Therefore, in our economy with idiosyncratic uncertainty, the steady-state credit spread depends on an additional component, compared to existing literature, and in particular to Benigno et al (2020), triggered by the anticipative motives that affect both types of agents when the steady state is unequal, and captured by Γ_s and Γ_b, i.e. the expected change in the marginal utility of consumption respectively for savers and borrowers. Thereby, $\Gamma > 1$ implies a positive consumption risk for savers, who therefore want to save more for precautionary reasons, compared to a steady state with $\Gamma = 1$. This implies $\Gamma_s > 1$ and a downward pressure on the return on deposits, by equation (A.65): $\beta(1+\bar{\imath}^R) < 1+\pi^*$. In addition, $\Gamma > 1$ also implies better prospects for the consumption of the borrowers (since they face a positive probability of turning savers) who therefore want to borrow more for anticipative reasons, compared to an equal steady state. This implies $\Gamma_b < 1$ and an upward pressure on the return on private bonds, by (A.66): $\beta(1+\bar{\imath}^B) > (1+\bar{\epsilon})^{-1}(1+\pi^*)$. As a result, if $\Gamma > 1$ the steady state exhibits a positive credit spread (and thus a binding leverage constraint) despite equal discount factors, provided $\bar{\epsilon}$ is sufficiently small relative to Γ.

B Proofs of Propositions

B.1 Proposition 1

Proof. Under the assumptions of Proposition 1, a first-order approximation of equations (A.45)–(A.46) yields

$$c_{bt} = \gamma_b E_t c_{bt+1} + (1-\gamma_b) E_t c_{st+1} - \sigma^{-1}(\hat{\imath}_t^B - E_t \pi_{t+1} + E_t \Delta \hat{\xi}_{t+1}) \tag{B.70}$$

$$c_{st} = \gamma_s E_t c_{st+1} + (1-\gamma_s) E_t c_{bt+1} - \sigma^{-1}(\hat{\imath}_t^R - E_t \pi_{t+1} + E_t \Delta \hat{\xi}_{t+1}) \tag{B.71}$$

where $\gamma_s \equiv p_s/\Gamma_s$ and $\gamma_b \equiv p_b/\Gamma_b$, with $0 < \gamma_s, \gamma_b < 1$ and we have used $\hat{\imath}_t^R = \hat{\imath}_t^D$, as implied by a first-order approximation of (A.47), and the result that, since $f(1) = 0$, there is no role for individual debt levels in the equilibrium interest rate on private assets, implying $\epsilon_t = \bar{\epsilon}$ for all t. Taking the difference of the above equations results in the equation (9) describing the dynamics of consumption inequality $\omega_t \equiv c_{st} - c_{bt}$:

$$\omega_t = (\gamma_s + \gamma_b - 1) E_t \omega_{t+1} + \sigma^{-1} \left(\hat{\imath}_t^B - \hat{\imath}_t^R \right). \tag{B.72}$$

Using a first-order approximation of the resource constraint (A.48) with equations (B.71)–(B.70) and the definition of output gap ($x \equiv y - y^*$) and consumption inequality yields equation (8):

$$\begin{aligned} x_t = E_t x_{t+1} &- \left[(1-z)(1-\gamma_s) - z(1-\gamma_b)\right] E_t \omega_{t+1} \\ &- \sigma^{-1}(\hat{\imath}_t^R - E_t \hat{\pi}_{t+1} + E_t \Delta \hat{\xi}_{t+1}) + E_t \Delta y_{t+1}^* - z\sigma^{-1}(\hat{\imath}_t^B - \hat{\imath}_t^R), \end{aligned} \tag{B.73}$$

where $y_t^* \equiv \frac{1+\varphi}{\sigma+\varphi} a_t$ denotes the potential level of output arising under flexible prices. Note that in the case of an equal steady-state, $\gamma_s = p_s$ and $\gamma_b = p_b$, implying that the coefficient in square brackets goes to zero by the restriction $(1-z)(1-s) = z(1-b)$. Accordingly, an unequal steady state implies an additional and separate role for consumption inequality in the dynamics of aggregate output, beyond the one implied by the private interest rate. Moreover, since $\Gamma > 1$ implies $\Gamma_s > 1$ and $\Gamma_b < 1$, it follows that $\gamma_s < p_s$, $\gamma_b > p_b$, and $\gamma \equiv (1-z)(1-\gamma_s) - z(1-\gamma_b) > 0$.

A first-order approximation of the borrower's budget constraint (A.44), using the labor supply equation and the production function implies equation (7). Using the latter, the resource constraint and the definition of consumption inequality then yields equation (10). A first-order approximation of (A.57), which uses Assumption 1, implies the equilibrium value of private debt (11). A first-order approximation of the supply block (A.49)–(A.52) finally yields the NKPC (12). ■

B.2 Proposition 4.

Proof. Consider the system of equations describing the private-sector equilibrium, which we can obtain using equation (10) into (13),

$$x_t = \Phi E_t x_{t+1} - \sigma_x^{-1}(\hat{\imath}_t^R - E_t\hat{\pi}_{t+1} - r_t^*) - \delta E_t\{\Delta\hat{u}_{t+1} - \Delta u_{t+1}^*\} + z^{-1}\delta(1-\gamma_s)E_t\{\hat{u}_{t+1} - u_{t+1}^*\} \quad (B.74)$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa x_t. \quad (B.75)$$

and the following monetary-policy rules

$$\hat{\imath}_t^R = r_t^* + \phi_\pi\hat{\pi}_t + \phi_x x_t \quad (B.76)$$

$$\hat{u}_t = u_t^* - \psi_\pi\hat{\pi}_t - \psi_x x_t. \quad (B.77)$$

Substituting the policy rules in the IS equation, we can write the system more compactly in matrix form as

$$\mathbf{A}\begin{bmatrix} x_t \\ \hat{\pi}_t \end{bmatrix} = \mathbf{B}\begin{bmatrix} E_t x_{t+1} \\ E_t\hat{\pi}_{t+1} \end{bmatrix}, \quad (B.78)$$

where let

$$\mathbf{A} \equiv \begin{bmatrix} 1 + \sigma_x^{-1}\phi_x + \delta\psi_x & \sigma_x^{-1}\phi_\pi + \delta\psi_\pi \\ -\kappa & 1 \end{bmatrix}$$

and

$$\mathbf{B} \equiv \begin{bmatrix} \Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x & \sigma_x^{-1} + z^{-1}\delta(\gamma_s + z - 1)\psi_\pi \\ 0 & \beta \end{bmatrix}.$$

System (B.78) admits $x_t = \hat{\pi}_t = 0$ for all t as a (locally) unique solution if and only if the two eigenvalues of matrix $\mathbf{D} \equiv \mathbf{B}^{-1}\mathbf{A}$ are both outside the unit circle, where

$$\mathbf{D} = (\det\mathbf{B})^{-1}\begin{bmatrix} \mathbf{d}_{11} & \mathbf{d}_{12} \\ \mathbf{d}_{21} & \mathbf{d}_{22} \end{bmatrix}, \quad (B.79)$$

with

$$\det\mathbf{B} = \beta(\Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x) \quad (B.80)$$

and

$$\mathbf{d}_{11} \equiv \beta(1 + \sigma_x^{-1}\phi_x + \delta\psi_x) + \kappa(\sigma_x^{-1} + z^{-1}\delta(\gamma_s + z - 1)\psi_\pi) \tag{B.81}$$

$$\mathbf{d}_{12} \equiv \beta(\sigma_x^{-1}\phi_\pi + \delta\psi_\pi) - (\sigma_x^{-1} + z^{-1}\delta(\gamma_s + z - 1)\psi_\pi) \tag{B.82}$$

$$\mathbf{d}_{21} \equiv -\kappa(\Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x) \tag{B.83}$$

$$\mathbf{d}_{22} \equiv \Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x. \tag{B.84}$$

As proved in Woodford (2003), among others, this condition is satisfied if all of the following holds

$$i) \det \mathbf{D} > 1 \qquad ii) \det \mathbf{D} - \text{tr } \mathbf{D} > -1 \qquad iii) \det \mathbf{D} + \text{tr } \mathbf{D} > -1. \tag{B.85}$$

Consider now that $\det \mathbf{D} = (\det \mathbf{B})^{-1} \det \mathbf{A}$, with

$$\det \mathbf{A} = 1 + \sigma_x^{-1}\phi_x + \delta\psi_x + \kappa(\sigma_x^{-1}\phi_\pi + \delta\psi_\pi).$$

Accordingly, condition B.85.ii) can be written as

$$\det \mathbf{A} - (\mathbf{d}_{11} + \mathbf{d}_{22}) > -\det \mathbf{B}$$

and it requires

$$\begin{aligned} 1 + \sigma_x^{-1}\phi_x + \delta\psi_x + \kappa(\sigma_x^{-1}\phi_\pi + \delta\psi_\pi) - \beta(1 + \sigma_x^{-1}\phi_x + \delta\psi_x) \\ - \kappa(\sigma_x^{-1} + z^{-1}\delta(\gamma_s + z - 1)\psi_\pi) - \Phi - z^{-1}\delta(\gamma_s + z - 1)\psi_x > -\beta(\Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x), \end{aligned}$$

which, after some algebra, yields condition (23):

$$\sigma_x^{-1}\Big[(1-\beta)\phi_x + \kappa(\phi_\pi - 1)\Big] + z^{-1}\delta(1-\gamma_s)\Big[(1-\beta)\psi_x + \kappa\psi_\pi\Big] > (1-\beta)(\Phi - 1). \tag{B.86}$$

Moreover, condition B.85.i) requires

$$\sigma_x^{-1}\phi_x + \delta\Big[1 - \beta + z^{-1}\beta(1-\gamma_s)\Big]\psi_x + \kappa(\sigma_x^{-1}\phi_\pi + \delta\psi_\pi) > \beta\Phi - 1, \tag{B.87}$$

which is always satisfied for $\Phi \leq \beta^{-1}$ and it is generally implied by (B.86) also for $\Phi > \beta^{-1}$, as long as Φ is not too large, in which case condition (B.87) becomes necessary and (B.86) is implied.

Finally, condition B.85.iii) requires

$$\begin{aligned} 1 + \sigma_x^{-1}\phi_x + \delta\psi_x + \kappa(\sigma_x^{-1}\phi_\pi + \delta\psi_\pi) + \beta(1 + \sigma_x^{-1}\phi_x + \delta\psi_x) \\ + \kappa(\sigma_x^{-1} + z^{-1}\delta(\gamma_s + z - 1)\psi_\pi) + \Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x > -\beta(\Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x), \end{aligned}$$

which is always satisfied for non-negative response coefficients ϕ's and ψ's. As a consequence, for

Φ not too large, the equilibrium is determinate if and only if condition (23) is satisfied. ■

B.3 Proposition 8

Proof. Under the assumptions of Proposition 8, a first-order approximation of equation (A.10) delivers (68):

$$\hat{\imath}_t^B = \hat{\imath}_t^R + \eta(\hat{b}_t - \hat{\theta}_t - \hat{u}_t), \tag{B.88}$$

once we define

$$\eta \equiv \frac{1}{1+F(1,1)}\frac{\bar{Y}}{\bar{n}\bar{\theta}}\Big(F_1(1,1)+F_2(1,1)\Big). \tag{B.89}$$

A first-order approximation of equations (A.46) and (A.60) yields equation (B.71) and

$$c_{bt} = \gamma_b E_t c_{bt+1} + (1-\gamma_b)E_t c_{st+1} - \sigma^{-1}(\hat{\imath}_t^B - E_t\pi_{t+1} + E_t\Delta\hat{\xi}_{t+1}) - \sigma^{-1}\hat{\epsilon}_t \tag{B.90}$$

where $\hat{\epsilon}_t \equiv \log\frac{1+\epsilon_t}{1+\bar{\epsilon}}$ follows from approximating equation (A.61):

$$\hat{\epsilon}_t = \eta_\epsilon\left(\hat{b}_t - \hat{\theta}_t - \hat{u}_t\right) + v\left(\hat{b}_t - \bar{b}_u\hat{\theta}_t\right) \tag{B.91}$$

and where we defined

$$\eta_\epsilon \equiv \frac{1}{1+F(1,1)}\frac{\bar{Y}/\bar{n}\bar{\theta}}{1+\bar{\epsilon}}\Big(F_{21}(1,1)+F_{22}(1,1)\Big) - \frac{\bar{\epsilon}}{1+\bar{\epsilon}}\eta \tag{B.92}$$

$$v \equiv \frac{\bar{\epsilon}}{1+\bar{\epsilon}}\bar{b}_y^{-1} \tag{B.93}$$

and the ratios $\bar{b}_y \equiv \bar{b}/\bar{Y}$ and $\bar{b}_u \equiv \bar{b}/\bar{b}^f \geq 1$. Throughout, we use the notation $F_i(\cdot,\cdot)$ to denote the first derivative of function $F(\cdot,\cdot)$ with respect to its i^{th} argument, and $F_{ij}(\cdot,\cdot)$ to denote the second (cross-)derivative of $F(\cdot,\cdot)$ with respect to its i^{th} and j^{th} arguments.

Using (B.71), (B.90), (B.91), (72) and a first-order approximation of the resource constraint yields equation (66):

$$x_t = E_t x_{t+1} - \sigma^{-1}(\hat{\imath}_t^R - E_t\hat{\pi}_{t+1} - r_t^*) - \gamma E_t\omega_{t+1} - z\sigma^{-1}\Big[\eta^{-1}\varrho(\hat{\imath}_t^B - \hat{\imath}_t^R) + v(\hat{b}_t - \bar{b}_u\hat{\theta}_t)\Big], \tag{B.94}$$

where we defined $\varrho \equiv \eta + \eta_\epsilon$, while taking the difference of (B.71) and (B.90) delivers equation (67).

Finally, a first-order approximation of the borrower's budget constraint (A.59), using the labor supply equation and the production function implies equation (69), while approximating the supply block (A.49)–(A.52) yields the NKPC (70). ■

C The Welfare-Based Monetary-Policy Loss Function

In this section we provide details on the derivation of equation (5). To study the normative implications of idiosyncratic uncertainty for monetary policy in our baseline economy with heterogeneous households, we are interested in the Ramsey policy that maximizes the expected social welfare

$$\mathcal{W}_{t_0} \equiv E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[(1-\tilde{z}) \Big(U(C_{s,t}) - V(L_{s,t}) \Big) + \tilde{z} \Big(U(C_{b,t}) - V(L_{b,t}) \Big) \right] \right\}, \tag{C.95}$$

for some weight $\tilde{z}$, with $0 < \tilde{z} < 1$. To evaluate the implied tradeoffs and derive the optimal monetary policy, we can use a purely quadratic loss function deriving from a second-order approximation of (C.95) around a socially optimal allocation. Such allocation is consistent with the solution of the Ramsey problem that maximizes (C.95) subject to the resource and technological constraint

$$A_t L_{s,t}^{1-z} L_{b,t}^{z} = Y_t = (1-z) C_{s,t} + z C_{b,t}. \tag{C.96}$$

The solution of this problem requires an appropriate cross-sectional distribution of steady-state consumption, given the welfare weight $\tilde{z}$:

$$\frac{1-\tilde{z}}{\tilde{z}} = \frac{1-z}{z} \frac{U_c(\bar{C}_b)}{U_c(\bar{C}_s)} \tag{C.97}$$

and an intratemporal efficiency condition for each type of agent:

$$\frac{V_L(\bar{L}_s)\bar{L}_s}{U_c(\bar{C}_s)} = \frac{V_L(\bar{L}_b)\bar{L}_b}{U_c(\bar{C}_b)} = \bar{Y}. \tag{C.98}$$

For a given long-run consumption inequality in the decentralized allocation, an appropriate welfare weight $\tilde{z}$ makes sure that the steady state satisfies condition (C.97), while the optimal employment subsidy τ^*—if the long-run employment risk is small enough—ensures negligible deviations from condition (C.98).[43] Thus, a quadratic Taylor expansion of (C.95) around a steady state that satisfies these two restrictions is a valid second-order approximation of expected social welfare that can be evaluated using only first-order-approximated equilibrium conditions. The maximum social welfare (C.95) consistent with the resource and technological constraints (C.96) requires:

$$(1-\tilde{z}) U_c(\bar{C}_s) = (1-z)\bar{\lambda} \tag{C.99}$$

$$\tilde{z} U_c(\bar{C}_b) = z\bar{\lambda} \tag{C.100}$$

$$(1-\tilde{z}) V_l(\bar{L}_s) = (1-z)\bar{\lambda} \frac{\bar{Y}}{\bar{L}_s} \tag{C.101}$$

$$\tilde{z} V_l(\bar{L}_b) = z\bar{\lambda} \frac{\bar{Y}}{\bar{L}_b} \tag{C.102}$$

where $\bar{\lambda}$ is the Lagrange multiplier on the contraint (C.96), evaluated at the steady state.

[43] The steady state is indeed distorted for both monopolistic competition and unemployment risk, see (A.62). However, a sufficient condition for the linear-quadratic approach to be valid is that the deviation from condition (C.98) be small enough (see Woodford, 2003, Ch. 6). Accordingly, for the normative analysis—and the numerical simulations—we assume, without loss of generality, that $1 - g(0) = \mathcal{O}(\|\mathbf{x}\|^2)$, with $0 < g(0) < 1$ and $\mathbf{x} \equiv (\hat{\xi}, y^*, \hat{\theta})$. An equivalent alternative would be to assume type-specific optimal employment subsidies that correct both distortions.

A second-order approximation of the social welfare (C.95) around this steady state reads:

$$\begin{aligned} U_t = \bar{U} + (1-\tilde{z}) & \left[U_c(\bar{C}_s)(C_{s,t} - \bar{C}_s) + \frac{1}{2} U_{cc}(\bar{C}_s)(C_{s,t} - \bar{C}_s)^2 \right] \\ & + \tilde{z} \left[U_c(\bar{C}_b)(C_{b,t} - \bar{C}_b) + \frac{1}{2} U_{cc}(\bar{C}_b)(C_{b,t} - \bar{C}_b)^2 \right] \\ & - (1-\tilde{z}) \left[V_l(\bar{L}_s)(L_{s,t} - \bar{L}_s) + \frac{1}{2} V_{ll}(\bar{L}_s)(L_{s,t} - \bar{L}_s)^2 \right] \\ & - \tilde{z} \left[V_l(\bar{L}_b)(L_{b,t} - \bar{L}_b) + \frac{1}{2} V_{ll}(\bar{L}_b)(L_{b,t} - \bar{L}_b)^2 \right] + \mathcal{O}(\|\mathbf{x}\|)^3, \end{aligned} \tag{C.103}$$

where an upper-bar denotes a variable in the steady state and the term $\mathcal{O}(\|\mathbf{x}\|)^3$ collects terms in the expansions that are of an order higher than two.

Using conditions (C.99)–(C.102) in equation (C.103), the latter can be cast in the form

$$\begin{aligned} U_t = \bar{U} + (1-z)\bar{\lambda} & \left[(C_{s,t} - \bar{C}_s) + \frac{1}{2} \frac{U_{cc}(\bar{C}_s)}{U_c(\bar{C}_s)} (C_{s,t} - \bar{C}_s)^2 \right] \\ & + z\bar{\lambda} \left[(C_{b,t} - \bar{C}_b) + \frac{1}{2} \frac{U_{cc}(\bar{C}_b)}{U_c(\bar{C}_b)} (C_{b,t} - \bar{C}_b)^2 \right] \\ & - (1-z)\bar{\lambda}\frac{\bar{Y}}{\bar{L}_s} \left[(L_{s,t} - \bar{L}_s) + \frac{1}{2} \frac{V_{ll}(\bar{L}_s)}{V_l(\bar{L}_s)} (L_{s,t} - \bar{L}_s)^2 \right] \\ & - z\bar{\lambda}\frac{\bar{Y}}{\bar{L}_b} \left[(L_{b,t} - \bar{L}_b) + \frac{1}{2} \frac{V_{ll}(\bar{L}_b)}{V_l(\bar{L}_b)} (L_{b,t} - \bar{L}_b)^2 \right] + \mathcal{O}(\|\mathbf{x}\|)^3. \end{aligned} \tag{C.104}$$

Now define $x_t \equiv \ln(X_t/\bar{X})$, for $X = Y,\ L,\ A$, which implies

$$\frac{X_t - \bar{X}}{\bar{X}} = x_t + \frac{1}{2}x_t^2 + \mathcal{O}(\|\mathbf{x}\|)^3. \tag{C.105}$$

Moreover, define $c_{s,t} \equiv (C_{s,t} - \bar{C}_s)/\bar{Y}$ and $c_{b,t} \equiv (C_{b,t} - \bar{C}_b)/\bar{Y}$, which imply, together with the resource constraint (A.48):

$$(1-z)c_{s,t} + zc_{b,t} = y_t + \frac{1}{2}y_t^2 + \mathcal{O}(\|\mathbf{x}\|)^3. \tag{C.106}$$

Using the above in (C.104) we obtain

$$\begin{aligned} U_t = \bar{U} + \bar{\lambda}\bar{Y} & \left[y_t + \frac{1}{2}y_t^2 \right] - \frac{1}{2}\bar{\lambda}\bar{Y}\sigma \left[(1-z)c_{s,t}^2 + zc_{b,t}^2 \right] \\ & - (1-z)\bar{\lambda}\bar{Y} \left[l_{s,t} + \frac{1}{2}(1+\varphi)l_{s,t}^2 \right] - z\bar{\lambda}\bar{Y} \left[l_{b,t} + \frac{1}{2}(1+\varphi)l_{b,t}^2 \right] + \mathcal{O}(\|\mathbf{x}\|)^3 \end{aligned} \tag{C.107}$$

where we used $U_{cc}(\bar{C}^s)/U_c(\bar{C}^s) = U_{cc}(\bar{C}^b)/U_c(\bar{C}^b) = -v$, $\bar{L}_s V_{ll}(\bar{L}_s)/V_l(\bar{L}_s) = \bar{L}_b V_{ll}(\bar{L}_b)/V_l(\bar{L}_b) = \varphi$ and $\sigma \equiv v\bar{Y}$. Now note that the aggregate production function $Y_t\Delta_t^p = A_t L_t = A_t L_{s,t}^{1-z} L_{b,t}^z$ implies that the following holds exactly:

$$y_t = (1-z)l_{s,t} + zl_{b,t} + a_t - \ln \Delta_t^p,$$

which allows us to simplify the linear terms in equation (C.107), and write:

$$\frac{1}{2}\frac{U_t - \bar{U}}{\bar{\lambda}\bar{Y}} = y_t^2 - \sigma[(1-z)c_{s,t}^2 + zc_{b,t}^2] - (1+\varphi)[(1-z)l_{s,t}^2 + zl_{b,t}^2] - \ln \Delta_t^p + \text{t.i.p.} + \mathcal{O}(\|\mathbf{x}\|)^3 \quad \text{(C.108)}$$

where "t.i.p." collects terms independent of policy. To evaluate the second-order terms in the equation above, we can use a first-order approximation of the resource constraint and of the aggregate production function:

$$c_{s,t} = y_t + z\omega_t \quad \text{(C.109)}$$

$$c_{b,t} = y_t - (1-z)\omega_t \quad \text{(C.110)}$$

$$l_{s,t} = y_t - a_t + zl_{R,t} \quad \text{(C.111)}$$

$$l_{b,t} = y_t - a_t - (1-z)l_{R,t}, \quad \text{(C.112)}$$

where $\omega_t \equiv c_{s,t} - c_{b,t}$ and $l_{R,t} \equiv l_{s,t} - l_{b,t}$

Using the above in equation (C.108), after some algebra we can write:

$$-\frac{1}{2}\frac{U_t - \bar{U}}{\bar{\lambda}\bar{Y}} = (\varphi+\sigma)x_t^2 + z(1-z)\Big(\sigma\omega_t^2 + (1+\varphi)l_{R,t}^2\Big) + \ln \Delta_t^p + \text{t.i.p.} + \mathcal{O}(\|\mathbf{x}\|)^3, \quad \text{(C.113)}$$

where $x_t \equiv y_t - y_t^*$ and $y_t^* \equiv \frac{1+\varphi}{\sigma+\varphi}a_t$.

Making use of the familiar result about the relative-price dispersion Δ_t^p,

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \ln \Delta_t^p = \frac{1}{2}\frac{1+\mu}{\mu}\frac{\vartheta}{(1-\vartheta)(1-\vartheta\beta)}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\pi_t^2 + \text{t.i.p} + \mathcal{O}(\|\mathbf{x}\|)^3 \quad \text{(C.114)}$$

we finally obtain:

$$\begin{aligned}\mathcal{L}_{t_0} &\equiv -\frac{1}{2}E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\frac{U_t-\bar{U}}{\bar{\lambda}\bar{Y}}\right)\right\} \\ &= \frac{\sigma+\varphi}{2}E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(x_t^2 + \lambda_\pi\hat{\pi}^2 + \lambda_c\omega_t^2 + \lambda_c\frac{1+\varphi}{\sigma}l_{R,t}^2\right)\right\}, \quad \text{(C.115)}\end{aligned}$$

which ignores terms independent of policy and of higher order, and where

$$\lambda_\pi \equiv \frac{1+\mu}{\mu\kappa} \qquad \lambda_c \equiv \frac{z(1-z)\sigma}{\sigma+\varphi}$$

Finally, recall the equilibrium condition in the labor market, i.e.

$$\varpi_t \frac{L_{s,t}^{1+\varphi}}{v\exp(-vC_{s,t})} = \frac{L_{b,t}^{1+\varphi}}{v\exp(-vC_{b,t})} \tag{C.116}$$

and note that a first-order approximation implies

$$l_{R,t} = -\frac{\sigma}{1+\varphi}\omega_t - \frac{\varpi_x}{1+\varphi}x_t, \tag{C.117}$$

which allows us to recast the loss function in the form of equation (5):

$$\mathcal{L}_{t_0} = \frac{\sigma+\varphi}{2}E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(x_t^2 + \lambda_\pi\hat{\pi}^2 + \lambda_c\omega_t^2 + \lambda_l\left(\sigma\omega_t + \varpi_x x_t\right)^2\right)\right\}, \tag{C.118}$$

where

$$\lambda_\pi \equiv \frac{1+\mu}{\mu\kappa} \qquad \lambda_c \equiv \frac{z(1-z)\sigma}{\sigma+\varphi} \qquad \lambda_l \equiv \frac{z(1-z)}{(\sigma+\varphi)(1+\varphi)}.$$

D Optimal policy under full commitment

The optimal policy under full commitment minimizes the loss (5)

$$\mathcal{L}_{t_0} = \frac{\sigma+\varphi}{2}E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(x_t^2 + \lambda_\pi\hat{\pi}_t^2 + \lambda_c\omega_t^2 + \lambda_l\left(\sigma\omega_t + \varpi_x x_t\right)^2\right)\right\}, \tag{D.119}$$

subject to the system (66)–(70) and the ZLB constraint on the policy rate:

$$x_t = E_t x_{t+1} - \sigma^{-1}(\hat{\imath}_t^R - E_t\hat{\pi}_{t+1} - r_t^*) - \gamma E_t\omega_{t+1} - z\sigma^{-1}\left[\eta^{-1}\varrho(\hat{\imath}_t^B - \hat{\imath}_t^R) + v(\hat{b}_t - \bar{b}_u\hat{\theta}_t)\right], \tag{D.120}$$

$$\omega_t = \left(\gamma_s + \gamma_b - 1\right)E_t\omega_{t+1} + \sigma^{-1}\left[\eta^{-1}\varrho(\hat{\imath}_t^B - \hat{\imath}_t^R) + v(\hat{b}_t - \bar{b}_u\hat{\theta}_t)\right], \tag{D.121}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa_\varpi x_t, \tag{D.122}$$

$$\hat{\imath}_t^R \geq -\bar{\imath}^R, \tag{D.123}$$

$$\hat{\imath}_t^B = \hat{\imath}_t^R + \eta(\hat{b}_t - \hat{\theta}_t - \hat{u}_t), \tag{D.124}$$

$$\begin{aligned}\beta\hat{b}_t = {} & \left(\gamma_s + \gamma_b - \Gamma_s^{-1}\right)\left[\hat{b}_{t-1} + \bar{b}_y(\hat{\imath}_{t-1}^R - \hat{\pi}_t)\right] + \gamma_b\bar{b}_y(\hat{\imath}_{t-1}^B - \hat{\imath}_{t-1}^R) \\ & + z\beta\left[(1-\varpi)y_t^* - (\chi_\varpi - 1)x_t - (1-z)\omega_t\right].\end{aligned} \tag{D.125}$$

Using $\mu_{j,t}$, for $j = 1, ..., 6$ to denote the Lagrange multipliers on constraints (D.120)–(D.125), the first-order conditions of the optimal policy problem conditional on $\hat{u}_t = 0$ for all t, with respect to $x_t, \omega_t, \hat{\pi}_t, \hat{\imath}_t^R, \hat{\imath}_t^B$, and $\hat{b}_t$, together with the slackness condition, are:

$$0 = x_t + \lambda_l \varpi_x(\sigma\omega_t + \varpi_x x_t) + \mu_{1,t} - \beta^{-1}\mu_{1,t-1} - \kappa_\varpi \mu_{3,t} + z\beta(\chi_\varpi - 1)\mu_{6,t} \quad \text{(D.126)}$$

$$0 = \lambda_c \omega_t + \lambda_l \sigma(\sigma\omega_t + \varpi_x x_t) + \mu_{2,t} - \beta^{-1}(\gamma_s + \gamma_b - 1)\mu_{2,t-1} + \beta^{-1}\gamma\mu_{1,t-1} + z(1-z)\beta\mu_{6,t} \quad \text{(D.127)}$$

$$0 = \lambda_\pi \pi_t + \mu_{3,t} - \mu_{3,t-1} - \beta^{-1}\sigma^{-1}\mu_{1,t-1} + (\gamma_s + \gamma_b - \Gamma_s^{-1})\bar{b}_y \mu_{6,t} \quad \text{(D.128)}$$

$$0 = \mu_{4,t} - \sigma^{-1}\left(\eta^{-1} z\varrho - 1\right)\mu_{1,t} + \sigma^{-1}\eta^{-1}\varrho\mu_{2,t} - \mu_{5,t} - \beta(\gamma_s - \Gamma_s^{-1})\bar{b}_y E_t \mu_{6,t+1} \quad \text{(D.129)}$$

$$0 = \sigma^{-1}\eta^{-1} z\varrho\mu_{1,t} - \sigma^{-1}\eta^{-1}\varrho\mu_{2,t} + \mu_{5,t} - \beta\gamma_b \bar{b}_y E_t \mu_{6,t+1} \quad \text{(D.130)}$$

$$0 = \sigma^{-1} z\upsilon\mu_{1,t} - \sigma^{-1}\upsilon\mu_{2,t} - \eta\mu_{5,t} + \beta\mu_{6,t} - \beta(\gamma_s + \gamma_b - \Gamma_s^{-1})E_t \mu_{6,t+1} \quad \text{(D.131)}$$

$$0 = \mu_{4,t}\left(\hat{\imath}_t^R + \bar{\imath}^R\right). \quad \text{(D.132)}$$

For the *unconditional* optimal policy problem, instead, they include the above equations plus the following condition, which relates to the optimal choice of the unconventional tool $\hat{u}_t$:

$$\mu_{5,t} = 0. \quad \text{(D.133)}$$

E Calibration

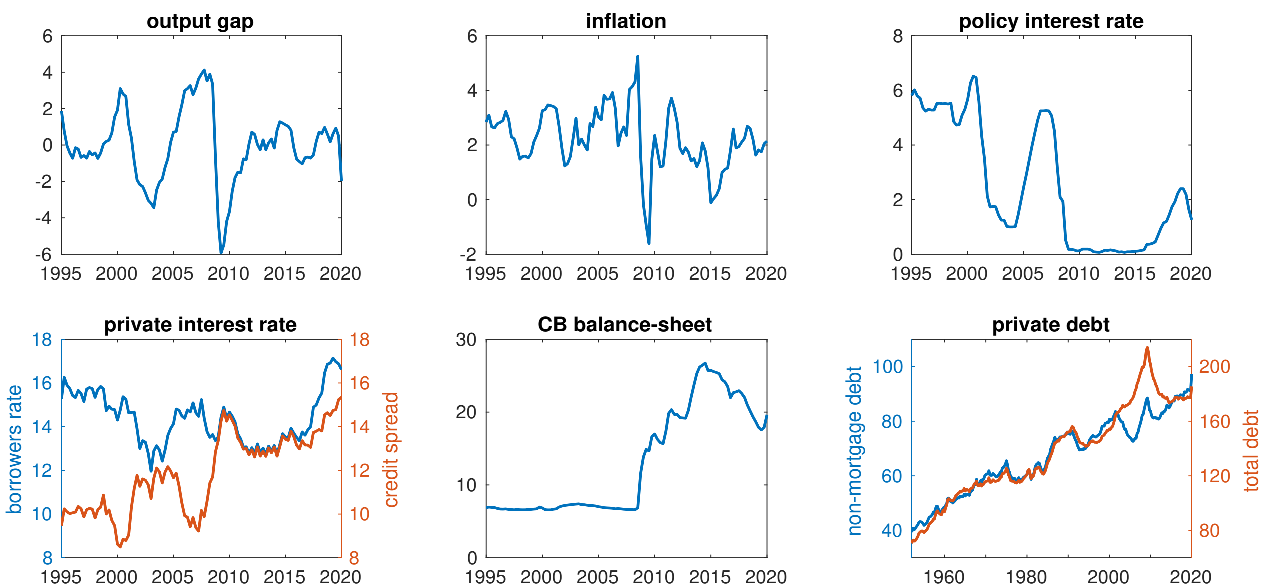

Figure 6: Evolution of variables of interest in the data. Last two panels are percentages of aggregate output, approximated as the sum of Personal Consumption Expenditures and Gross Private Domestic Investment.

Table 1: Calibrated and implied parameter values in the numerical illustrations

Parameter	Description	Value	Source or target
$\bar{Y}$	SS real output	1	Normalization
σ	Inverse of the IES in consumption	1	Convention
φ	Inverse of the Frisch elasticity	1	Convention
z	Share of constrained households	0.37	Bilbiie (2018)
p_s	Probability of remaining savers	0.96	Bilbiie, Primiceri and Tambalotti (2022)
π^*	Inflation target	0.005	Inflation target = 2% annual
β	Time discount factor	0.9772	Real interest rate = 1% annual
μ	Price markup	0.15	Rotemberg and Woodford (1997)
ϑ	Calvo parameter	0.85	Justiniano, Primiceri and Tambalotti (2013)
$\bar{b}$	SS private debt to GDP	3.4445	Private debt net of mortgages (2008q4) Source: FRED Economic Data
$\bar{u}$	SS CB balance sheet to GDP	0.4634	Source: FRED Economic Data (2008q4)
$\bar{n}$	SS banks net worth to GDP	0.4243	Total Bank Equity Capital (2008q4) Source: FRED Economic Data
ϖ_x	Cyclicality of unemployment risk	0.8	Okun's Law (Ball et al, 2017)
ν	Profits and tax share of borrowers	$= z$	Benigno, Eggertsson and Romei (2020)
$\bar{\epsilon}$	SS spread elasticity to indiv. debt	0.0190	Borrowers interest rate = 13.36% annual Interest Rate on Credit Card Plans (2008q4) Source: FRED Economic Data
η	Parameter in spread equation	0.12	Match the data
ϱ	Parameter in inequality equation	0.19	Match the data
p_b	Probability of remaining borrowers	0.9319	Implied by $z(1-p_b) = (1-z)(1-p_s)$
$\bar{\theta}$	SS leverage ratio to GDP	7.0264	Implied by $\bar{b} = \bar{\theta}\bar{n} + \bar{u}$ Reference: FRED Economic Data
υ	Parameter in inequality equation	0.0054	Implied by $\upsilon \equiv \frac{\bar{\epsilon}}{1+\bar{\epsilon}}\bar{b}^{-1}$

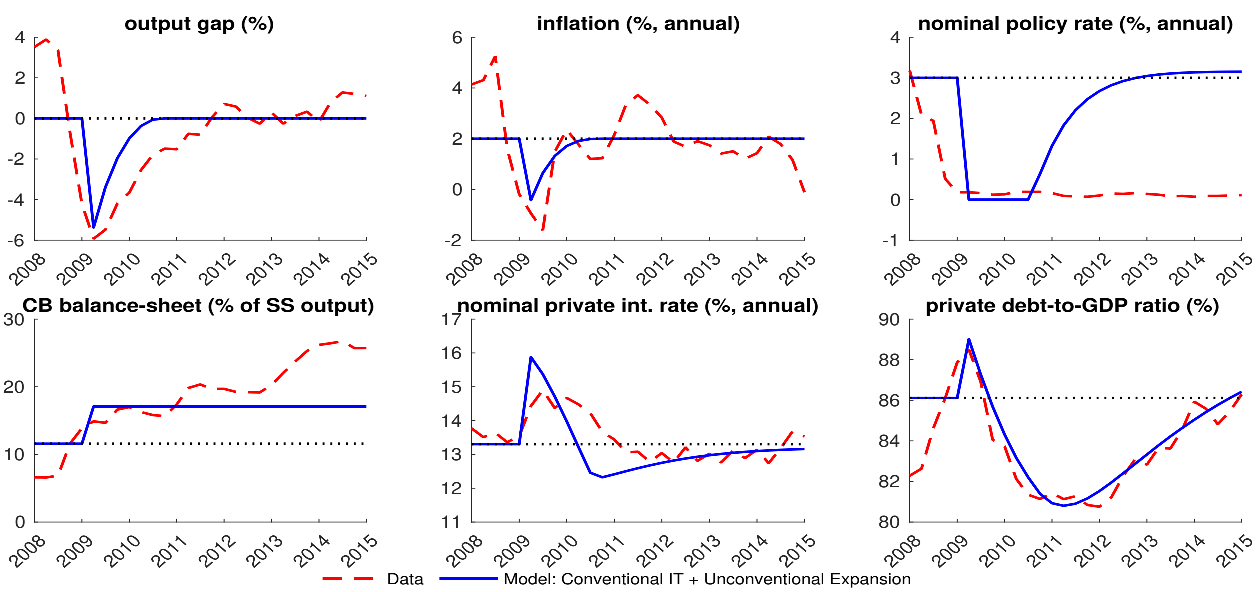

Figure 7: Model vs Data