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# The U-Shaped Growth Equation*

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**Abstract**

We present empirical evidence supporting a non-monotonic relationship between employment and productivity growth, using data from the US. We then study the theoretical implications of such U-shaped relation in a "Keynesian Growth" framework with heterogeneous innovation, which reconciles the market-size and opportunity-cost views of technological change. We show that a U-shaped growth equation can rationalize the existence of locally determinate equilibria with unemployment and low growth in a liquidity trap. We also show that growth policy incentivising exploratory research activities leading to radical innovation can both prevent the exposure to unemployment equilibria and help the economy escape them.

*JEL codes*: E32, E43, E52

*Keywords*: Endogenous Growth, Stagnation Trap, Jobless Growth, Hysteresis, Liquidity Traps, Heterogeneous innovation.

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# 1 Introduction

Schumpeter (1934) writes that recessions are times of creative destruction, during which new products and techniques are improved or developed and displace the old ones (Barro and Sala-i Martin, 2004). This view of counter-cyclical innovation found support in the ”opportunity cost” theory of innovation, according to which the foregone profits of investing capital or labor resources in technological or managerial improvements is lower during downturns.[1] This view of innovative activity has been challenged by some empirical works, which instead tend to favor the alternative ”market size” theory of innovation.[2] The latter states that product demand is the main driver of research and development efforts and, as a result, technological progress is procyclical. This theory is at the core of most modern endogenous-growth models, which therefore feature a procyclical innovation engine.

This paper reconciles these views within a unified theory of technological progress.

We start with an empirical analysis of the *growth equation*, i.e. the relation between employment and productivity drift that summarizes the supply-side of growth theory. The latter is implied to be positive under the ”market size” and negative under the ”opportunity cost” theories of innovation. We focus on the US economy during the period 1948-2019 and identify the growth-equation controlling for supply shocks, building on the approach first engineered for identification of the Phillips Curve (McLeay and Tenreyro, 2020). Our results show that neither a positive nor a negative growth equation is clearly supported by the data. We then allow the data to be free to speak, imposing a more flexible structural relation in the form of a threshold model, and let it choose discontinuity points in the employment-growth link. The model identifies the 20th percentile of the employment historical distribution as a significant nonlinearity point: when the economy operates in normal times and shocks are mild, employment and productivity move in the same direction, while when shocks push the economy below the identified threshold, the growth equation turns upside down. Building on that intuition, we then show that a U-shaped growth equation is a better characterization of the employment-growth link.

What can explain a U-shaped growth equation? The answer is in both the market size and opportunity cost theories, once allowing for heterogeneous innovation. We consider an economy in which technological progress is the result of interaction between radi-

[1] Aghion and Saint-Paul (1991), Aghion and Saint-Paul (1998b), Gali and Hammour (1992), Cooper and Haltiwanger (1993), Caballero and Hammour (1994), Hall (1991), Saint-Paul (1997), DeLong (1990), Canton and Uhlig (1999) and King and Robson (1989).

[2] See, among others, Griliches (1990) and Comin and Gertler (2006).

cal/exploratory and incremental/exploitative innovation.[3] Radical innovation includes types of inventions that create new fringes, new areas of knowledge, are more risk taking and produce higher expected payoffs. Incremental innovation instead deals with refinements of existing products, and thus generates lower expected profits. The opportunity cost of venturing into exploratory research—in terms of foregone profits that could have been achieved with incremental technological refinements—is lower during recessions. As a result, this type of activity will be countercyclical. Conversely, during an expansion, exploitation efforts become more appealing because the associated expected profits are higher (market size effect), hence incremental innovation will be procyclical. To keep the model tractable, we assume entrepreneurs are alike in their capability of performing incremental innovation, while have heterogeneous skills in engaging in radical innovation. Free-entry into research partitions entrepreneurs between radical (high ability) and incremental (low ability) innovators, so that both types of activities are undertaken at each point in time. The threshold is driven by the business cycle, with the share of radical and incremental innovators time varying. Given this setup, the growth equation summarizing the supply-side of the economy is U-shaped, with one margin dominating over the other depending on the business cycle: when shocks are small, the incremental margin dominates and growth is driven by the market size effect; when recessions are deep instead, the opportunity cost of conducting exploration activities is low, more and more radical innovations are created, and growth increases in unemployment. This mechanism is consistent with novel empirical evidence showing that firms shit toward exploration during contractions and exploitation during expansions, using nuanced measures of patent characteristics (Manso et al., 2023). Whether or not the opportunity cost margin acts as a built-in economic resilience mechanism depends on deep parameters and on the aggregate demand block of the economy, which we describe next.

We assume a Keynesian aggregate demand block: wages are sticky, the central bank follows a Taylor rule taking into account the zero lower bound and households face idiosyncratic unemployment risk, so that involuntary unemployment can arise and monetary policy is meaningful.

The interaction between the U-shaped growth equation and Keynesian aggregate demand gives rise to several implications.

First, multiple equilibria are possible. As long as shocks are small and the central bank is not constrained by the zero lower bound, the economy finds itself in a full-employment

[3] Henceforth, we will use as synonyms the terms radical, exploratory and horizontal to refer to innovation that leads to the creation of novel varieties, and incremental, exploitative and vertical to refer to innovation improving on existing varieties.

equilibrium: output and employment are at potential and growth is strong. When shocks are harsher and the central bank is constrained by the zero lower bound, a liquidity trap arises and two possible equilibria are admissible: the first one is a "stagnation trap", as in the seminal work of Benigno and Fornaro (2018) on which we build, with low-growth and low-employment; the second one is a "jobless growth" trap, with high-growth and low-employment. Thus our model can rationalize the persistent slumps of the Euro Area and Japan (Benigno and Fornaro, 2018) as well as the fast, but jobless, recovery of the US (Schmitt-Grohe and Uribe, 2017) in the post global financial crisis period.

Second, we show that the full employment and jobless growth equilibria are locally determinate, while the stagnation trap equilibrium—as in Benigno and Fornaro (2018)—is locally indeterminate. This has important implications. Structural shocks are the equilibrium selection device: deep recessions will fall in the basin of attraction of the jobless growth trap, while mild recessions into the one of full employment. Only sunspot shocks and animal spirits could yield the economy into a stagnation trap. Also, depending on deep parameters, the two "bad" equilibria could actually almost overlap: in that scenario we would get essentially two stagnation traps, one of which is locally determinate. This helps in better understanding the persistent slumps and liquidity traps observed in the Euro Area and Japan (stagnation traps), which instead would have required a substantial coordination in expectations to materialize.

Third, if the radical innovation margin is strong enough, it could prevent the occurence of bad equilibria altogether. However, if they occur, they are generally harsher (deeper and determinate) than in the case of a linear, upward-sloping, growth equation. This naturally leads to the discussion of policy implications, which is our fourth result. As far as monetary policy is concerned, we show that countercyclical radical innovation gives more room to dovish reaction of the central bank to employment deviations because the link between growth and employment on the positively sloped portion of the growth-equation is milder. In terms of industrial policies instead, a subsidy to exploratory research activities can prevent the formation of bad equilibria or steer the economy away from them.

**Related Literature.** Our work is related to several strands of the economic literature. We relate to works that provide a unified study of business-cycles and growth (Stadler, 1986, 1990; Stiglitz, 1993; Martin and Rogers, 1997; Fatàs, 2000; Comin and Gertler, 2006; Anzoategui et al., 2019; Bianchi et al., 2019; Garga and Singh, 2020; Benigno and Fornaro, 2018; Comin, 2009; Schmitt-Grohe and Uribe, 2017; Eggertsson et al., 2019; D'Amico, 2024)[4]. These works have either a market-size or an opportunity-cost growth engine in the background. We contribute to this literature by providing a framework in which heterogeneous innovation allows both engines to be at work, with the business cycle driving which one prevails at the aggregate level.

We contribute to the Keynesian endogenous growth literature (Benigno and Fornaro, 2018, Garga and Singh, 2020, Queralto, 2020, Anzoategui et al., 2019, Bianchi et al., 2019, Guerron-Quintana and Jinnai, 2019, Moran and Queralto, 2018), which introduces endogenous growth in business cycle models with nominal rigidities. Among these, we are most closely related to Benigno and Fornaro (2018) who show that an upward sloped growth equation can give rise to stagnation traps once interacted with a Keynesian aggregate demand block. However, their stagnation-trap equilibrium is locally indeterminate, whereby the transition from full employment to the unemployment steady state requires animal spirits and an appropriate coordination of private-sector expectations. Moreover, being unstable, small structural shocks can in general drive the system away from the trap. We complement Benigno and Fornaro (2018) seminal work in one key respect: we allow for both incremental and radical innovation, as opposed to only incremental. As a result, in line with the empirical evidence we provide, our growth equation is U-shaped, allowing for a third steady-state equilibrium to arise, one that is both associated to high unemployment and is locally determinate. In other words, our framework makes it possible for *stable* stagnation traps to arise, where the economy can end because of fundamental factors unrelated to animal spirits. This allows us to rationalize both the persistent slumps experienced in the Euro Area and Japan as well as the jobless-growth recovery of the US.

We are also related to Aghion and Saint-Paul (1991), Aghion and Saint-Paul (1998b), Gali and Hammour (1992), Cooper and Haltiwanger (1993), Caballero and Hammour (1994), Hall (1991), Saint-Paul (1997), DeLong (1990), Canton and Uhlig (1999) and King and Robson (1989) whom build models in which recessions have positive, cleansing effects, mainly trough an opportunity cost (or intertemporal substitution) effect. These works find empirical support in Davis and Haltiwanger (1992), Blanchard et al. (1990), Aghion and Saint-Paul (1998a), Bean (1990), Burnside et al. (1993), Basu and Fernald (1995), Dunne et al. (1996), Nickell et al. (2001) and Fatàs (2000) and more recently in Aghion et al. (2010; 2012), Fernald and Wang (2016) and Haltiwanger et al. (2021). Nonetheless, this empirical literature is not clear-cut on whether firms tend to innovate more or less during recession periods and it is therefore not possible to reject one theory in favor of the other. We complement this literature taking a different standpoint. Instead of debating on whether firms' innovation is pro- or counter-cyclical, we allow our framework

[4]For a complete historical review of this literature the reader is referred to Cerra et al. (2020).

to encompass both, with the understanding that pro- or counter-cyclicality will be driven by the business cycle. Moreover, our setup can shed some light on why the cleansing effects of recessions have troubles leaving visible effects on GDP growth: the presence of the zero lower bound on monetary policy can in fact make it more difficult to exploit the counter-cyclical side of innovation.

We are also closely related to Manso et al. (2023), who build on the intuition that experimentation with new ideas is the basis for innovation (Arrow, 1969) and that there is a clear distinction between exploration and exploitation (March, 1991). Using the distribution of number of patents per technology class and firm and a measure of similarity for the period 1958-2008, they show that firms pursue exploration during downturns and exploitation during booms. The opportunity cost of venturing into exploratory research leading to radical innovation—in terms of foregone expected profits related to incremental innovation—is lower during recessions. We complement this empirical evidence providing aggregate results for the link between productivity growth and employment over the business cycle. Also, in our theoretical framework we allow entrepreneurs to endogenously determine whether to engage in radical or incremental innovation, and show that this yields a U-shaped growth equation. Importantly, while Manso et al. (2023) contemplate the idea that countercyclical innovation can act as a built-in resilience mechanism that can attenuate the welfare losses associated with recessions, we show that this is not necessarily the case, as it depends on the particular characteristics of the R&D sector, the degree of wage rigidity, and the monetary policy stance.

Finally, we are related to the classics of endogenous growth (Segerstrom et al., 1990; Grossman and Helpman, 1991; Romer, 1990b; Aghion and Howitt, 1992) as we extend their framework introducing heterogeneous innovation, monetary policy and nominal rigidities.

**Structure.** The paper is organized as follows: Section 2 discusses empirical evidence in support of a U-shaped growth equation, Section 3 crafts a Keynesian growth model with heterogeneous innovation, Section 4 discusses the policy implications of our analysis and Section 5 concludes.

## 2 Empirical evidence

In the endogenous growth theoretical literature (see Barro and Sala-i Martin, 1997) a key relation links employment with productivity growth, describing the supply-side of the economy: the *growth equation*. This relation is typically implied to be monotonic and upward-sloping. The mechanism behind that is common to both expanding varieties models (à la Romer, 1990b) as well as to creative destruction ones (à la Aghion and Howitt, 1992): innovators invest in R&D to discover new varieties or better qualities, respectively in the expanding varieties or creative destruction frameworks, so as to acquire future expected monopolists profits related to the patented innovation. Since expected profits are an increasing function of employment (what in the literature is known as "market size effect"), this pins down a positive relationship between the level of employment and innovation intensity and, importantly, productivity growth. In endogenous growth models where the households side is defined in real terms, and thus with no monetary policy, the growth equation, together with an aggregate demand equation (i.e. the Euler equation of the household) determines the equilibrium level of growth and employment.

More recently, Benigno and Fornaro (2018) show that a supply-side of this kind, once interacting with a New Keynesian demand-side that explicitly accounts for an effective lower-bound on nominal interest rates, gives rise to multiple equilibria, one of which features low levels of employment and low productivity growth. In particular, in the specification of Benigno and Fornaro (2018), the optimal innovation intensity combined with the households' stochastic discount factor yields

$$(g_{t+1} - 1)\left(1 - \beta \mathbb{E}_t \left[ \left( \frac{c_t}{c_{t+1}} \right)^{\sigma} g_{t+1}^{-\sigma} \chi \gamma \varpi L_{t+1} \right] \right) = 0, \tag{2.1}$$

where $g_{t+1}$ is the (gross) productivity growth rate (between period $t$ and $t+1$), $c_t$ is consumption normalized by the productivity index, $L_{t+1}$ is employment, $\sigma$ is the inverse of the elasticity of intertemporal substitution and $\chi$, $\gamma$ and $\varpi$ are positive parameters. Equation (2.1) determines the optimal R&D effort by entrepreneurs and implies a monotonic upward-sloping relation between innovation-led growth $g_{t+1}$ and employment $L_{t+1}$.

An alternative link between growth and employment explored in the literature is indeed of the opposite sign, compared to standard growth equations. The "opportunity cost/intertemporal substitution" theory of innovation (Aghion and Saint-Paul, 1991, Aghion and Saint-Paul, 1998b, Gali and Hammour, 1992, Cooper and Haltiwanger, 1993, Caballero and Hammour, 1994, Hall, 1991, Saint-Paul, 1997, DeLong, 1990, Canton and Uhlig, 1999 and King and Robson, 1989) models recessions as good times to carry out innovation, for the opportunity cost drops and the expected value increases, in line with empirical evidence.[5] The main idea is that the foregone profits for investing resources in

[5] Davis and Haltiwanger (1992), Blanchard et al. (1990), Aghion and Saint-Paul (1998a), Bean (1990), Burnside et al. (1993), Basu and Fernald (1995), Dunne et al. (1996),Nickell et al. (2001) and Fatàs (2000) and more recently in Aghion et al. (2010; 2012) and Fernald and Wang (2016)

technological improvements or in managerial reorganizations are lower during depression phases, and the more so when recessions are deeper (Aghion and Saint-Paul, 1991). The complementary entrepreneurship literature emphasizes the ”cleansing” and ”sullying” effects that may be triggered during recessions (Haltiwanger et al., 2021). Taking together both the ”market size effect” and the ”opportunity cost/intertemporal substitution/cleansing and sullying” theories of innovation suggests that the growth equation may in fact be non-monotonic, with a slope driven by the business-cycle. This lays the foundations of our identification strategy which we will turn next.

## 2.1 Econometric approach

In this section we discuss our approach to identify and estimate the growth equation for the US economy. We will focus on the period 1984-2019.

### 2.1.1 Identification

Equilibria in an endogenous-growth model can be characterized using two *loci* in the employment-growth space: the growth equation, describing the supply side of the economy, and the aggregate-demand equation, describing the relation between growth and current aggregate demand. As noted in Lucas (1976) and more recently revived in McLeay and Tenreyro (2020), in the data we only observe intersection points of these two curves (or, more in general, equilibrium points), and supply and demand shocks might blur the identification of either curve.

The period we are focusing on had been mostly characterized by demand shocks (Smets and Wouters, 2007). As such, the latter move the aggregate demand equation and, if supply shocks are adequately controlled for, time variation in the data would then depict the growth equation we are interested in, allowing identification.

Given the massive structural change the US economy went through during the period under focus, especially in relation to how innovation is conceived and deployed, we don't expect the growth equation to have the same slope throughout. Rather, the link between employment and productivity growth should, if any, reflect that change. We will thus firstly naively estimate a linear model on the whole 1984-2019 period, and show that indeed the standard positively-sloped linear growth equation is not supported by the data. We will then identify a nonlinearity in the growth equation via threshold regression. To take this linearity into account, we will fit piecewise linear and quadratic models. Our results suggest that a more accurate representation of the link between employment and growth is given by a second order polynomial. The data identify a standard positive association between growth and employment during "normal" times, *i.e.* periods in which economic contractions are not particularly deep. Instead, when downturns provoke deep recessions, the data identify a downwardly sloped growth equation.

### 2.1.2 Model specification

Our approach is to estimate a linear approximation of equation (2.1) which involves a simple time-series regression of the form

$$g_t = \alpha + \beta L_t + \gamma' \mathbf{X}_t + \epsilon_t \tag{2.2}$$

where $g_t$ is the (log) productivity growth rate, $L_t$ the employment rate and $\mathbf{X}_t$ is a vector of controls. Our aim is not to explain productivity growth; rather, it is to let the data characterize the link between productivity growth and employment that describes the supply-side of the economy. To this end, our vector of controls $\mathbf{X}_t$ includes just the growth rate of consumption, as in the starting equation (2.1) and a proxy for supply shocks. We follow the recent literature on the estimation of the Phillips curve (Benigno and Eggertsson, 2023) and rely on "headline shocks" as indicators of supply shocks. In particular, we take the first principal component of three series: (i) the difference between headline and core CPI inflation; (ii) the difference between headline and personal consumption expenditures (PCE) price index inflation; (iii) the difference between the change in the import prices and the GDP deflator[6]. Our data are quarterly, spanning from 1984Q1 until 2019Q4, and described in the Appendix B.1.

**Table 1:** The U-shaped Growth Equation: estimation results

| | (1)<br>L. 1984–2019 | (2)<br>L. 1984–2000 | (3)<br>L. 1984–2007 | (4)<br>L. 2000–2019 | (5)<br>PWL 1984–2019 | (6)<br>Q. 1984–2019 |
|---|---|---|---|---|---|---|
| Empl | -0.013 | 0.016 | 0.019 | -0.018 | -0.295*** | -5.783*** |
| | (0.019) | (0.037) | (0.033) | (0.023) | (0.098) | (1.916) |
| Empl*Threshold | | | | | 0.345*** | |
| | | | | | (0.102) | |
| Empl2 | | | | | | 3.006*** |
| | | | | | | (0.998) |
| R2 | 0.053 | 0.098 | 0.022 | 0.059 | 0.171 | 0.111 |
| Controls | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 144 | 65 | 96 | 80 | 144 | 144 |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. "L." stands for linear regression specified in Model 2.2. "PWL" stands for piecewise linear regression specified in Model 2.4. "Q." stands for quadratic regression specified in Model 2.5.

[6] The individual time series together with the first principal component are plotted in Figure 5 in the Appendix C

Estimating Equation (2.2) on the whole time period yields a flat and not significant relationship between employment and productivity growth (Table 1, first column), with an overall poor fit (.05). We estimate model (2.2) in three alternative subsamples: (i) one spanning from 1984 til 2000, (ii) one for the Great Moderation period (1984-2007) and (iii) one for the last two decades (2000-2019). Results are reported in Table 1. None of these regressions identifies a significant growth equation in the data.

**Threshold regression.** Why is such a well known and key equation not supported by the data? Our hypothesis is that indeed, in the data there is evidence for *both* the ”market size” (Romer, 1990b, Aghion and Howitt, 1992, etc.) and ”cleansing” (Aghion and Saint-Paul, 1998b, Haltiwanger et al., 2021, etc.) effects of recessions at the same time, blurring the identification of the growth equation.

To test this hypothesis, we employ a threshold regression of the type

$$g_t = \alpha + \left(\beta_1 L_t + \gamma_1' \mathbf{X}_t\right) \mathbb{1}_{L_t \leq \bar{L}} + \left(\beta_2 L_t + \gamma_2' \mathbf{X}_t\right) \mathbb{1}_{L_t > \bar{L}} + \epsilon_t, \tag{2.3}$$

where $\mathbb{1}_{L_t \leq \bar{L}}$ is an indicator function taking value 1 when employment $L_t$ is below a certain threshold value $\bar{L}$, $\mathbf{X}_t$ is a vector of controls. The fact that we chose a very limited set of controls in estimating Model (2.2), allows us to keep the same vector for Model (2.3) without incurring in sample size limitations. The employment threshold $\bar{L}$ is our variable of interest, and it is estimated minimizing the Bayesian information criterion. In particular, the model tests for a series of candidate thresholds and for each candidate threshold, the data are split into two regions and linear regressions are fitted within each subregion. The model selects the optimal threshold that provides the best fit to the data, as indicated by the minimized Bayesian information criterion.

By using this approach, the threshold regression model endogenously determines the nonlinearity point, ensuring that the split is data-driven and not arbitrary. The found threshold signals a change in the structural relationship between growth and employment over the employment domain. Results are reported in Table 2. The model identifies .951 as a point of discontinuity in the relationship between growth and employment, which roughly corresponds to the 20th percentile of the employment distribution along the time domain. In particular, the growth equation is depicted to have a negative slope ($-.29$) when employment fall in the bottom 20% of its obsersations, and a positive one (.05) after that point, both of which are significant. The threshold regression then suggests that a good representation of the data would be a non-monotonic growth equation. This evidence suggests that it takes a large recession (one that pushes employment very deep)

**Table 2:** The U-shaped Growth Equation: threshold regression.

| | Employment | |
|---|---|---|
| | Region 1 $L_t \leq .951$ | Region 2 $L_t > .951$ |
| $L_t$ | -.294** (.098) | .05* (.031) |
| BIC | -1616.83 | |
| HQIC | -1630.93 | |
| Controls | ✓ | |
| $N$ | 144 | |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. The threshold is estimated minimizing the Bayesian information criteria.

to activate cost-opportunity or cleansing effects that might indeed stimulate growth.

**The nonlinear growth equation.** To further investigate the nonlinearity of the growth equation, we estimate a piecewise linear regression:

$$g_t = \alpha + \beta_1 L_t + \gamma_1' \mathbf{X}_t + \left(\beta_2 L_t + \gamma_2' \mathbf{X}_t\right) \mathbb{1}_{L_t > 95.1} + \epsilon_t, \tag{2.4}$$

where $\mathbb{1}_{L_t > 95.1}$ is an indicator function taking value 1 when employment $L_t$ is above the threshold value of 95.1 identified by the Threshold Regression Model (2.3) and $\mathbf{X}_t$ is the full vector of controls. Results are reported in the fifth column of Table 1. The coefficient on employment $L_t$ is negative and significant for the deep recessions subsample (-.295). The interaction term is positive, implying a coefficient on employment for observations above the 95.1 threshold of .05 (given by $.344 - .294$).

The last exercise conducted in this section relative to the growth-employment relationship, is to evaluate the performance of the quadratic regression model to fit the data relative to the linear one:

$$g_t = \alpha + \beta_1 L_t + \beta_2 L_t^2 + \gamma' \mathbf{X}_t + \epsilon_t. \tag{2.5}$$

The last column of Table 1 reports results of this exercise. The parameters governing the shape of the parabola in regression (2.5) depict a significant upward sloped parabola as a better way to represent the growth equation. Moreover, even if the $R^2$ are generally low for all regressions, as a result of the high dispersion in the data, the one associated with the quadratic model is more than 5 times higher than the one associated to the linear models. This evidence is consistent with the results from both the threshold regression and piecewise linear regression and it supports a nonlinear growth equation, where *both* market size effects (positive slope) *and* opportunity costs and cleansing effects (negative slope) are present.

## 2.2 Robustness

To test the robustness of our results we reestimate our models on two alternative measures of productivity growth: labour productivity of the business sector and total factor productivity, both from Fernald (2014). Results are reported in Tables 5, 6 and 7 in the Appendix C. The evidence on labour productivity of the business sector is almost identical, both qualitatively and quantitatively, to the baseline. In particular, there is no evidence of a linear and positevely sloped growth equation neither in the entire sample nor in subsamples, and a second order polynomial fitting growth to employment is statistically significant. Also, the employment threshold is the same as in the baseline. Total factor productivity instead points towards a mildly significant (at the 10% level) negative relation between growth and employment over the entire sample. However a U-shaped relation is significant at the 5% level and provides a better fit to the data ($R^2$ is .25 for the parabola against .23 for the linear model).

Another concern of the baseline specification is that structural changes (such as trend breaks in the demographic curve) or supply-related characteristics might hinder the identification of the growth equation. To account for this possibility, we ran all baseline regressions adding the growth rate of population, liabilities of the nonfinancial corporate business sector (as a proxy for credit availability to firms) and investment in R&D as controls. Results are reported in Table 8. Adding more controls leaves the results unchanged both qualitatively and quantitatively. However, now the U-shaped growth equation has a better fit to the data ($R^2$ is now .4 against .11 in the baseline). We can't check how the employment threshold is affected by this robustness because the inclusion of more controls increases the number of parameters to be estimated above what our datapoints allow to handle via a threshold regression.

One might be concerned that the supply shock we are using in the baseline specification is not properly capturing supply shocks. We check this claim estimating the baseline for all three additional proxies of supply shocks. Results are in Tables 9, 10, 11 respectively for CPI headline, PCE headline and import price shocks. Results are robust to this exercise and the employment threshold is the same as in the baseline.

A last exercise we perform is to estimate the baseline specifications with lagged employment as regressor. Indeed, in the theoretical model we develop below, the growth equation links current productivity growth to current employment. However, an alternative approach popular in the literature is to think of productivity growth as a sluggish process, hence requiring several cycles of innovation to materially move the technological frontier. We thus estimate our empirical models allowing current productivity to be affected by lagged values of employment. Results are reported in Tables 12,13, 15 and 14. The threshold regression Model 2.3 now identifies .953 as the threshold employment value for one quarter lagged employment, very close to the baseline of .951, however significant only at the 13% level. Instead, for two quarters lagged employment the threshold is .976, closer to the value found for Fernald (2014) TFP, however not significant. The linear regressions fail to identify a relationship between growth and employment, as in the baseline, for both one and two quarters lagged employment. In line with our main result, a quadratic specification instead fits the data well, with coefficients significant at the 5% level for one quarter and at the 10% level for two quarters lagged employment.

To summarise, in this section we have tested the robustness of our main finding that linear models do not provide a good characterization of the employment-growth link in the data, while a quadratic model does. We found that our results are robust to (i) changing dependent variables, (ii) adding more controls, (iii) allowing for different proxies of supply shocks and (iv) lagged values of employment.

# 3 Theoretical analysis

In this section we conceptualize the empirical evidence discussed above through the lens of a theoretical model of Keynesian growth, where we allow for heterogeneous innovation (radical and incremental) to drive technological change.

We consider a closed economy, populated by a continuum of unitary mass of households consuming a final good and supplying labor services, a continuum of unitary mass of perfectly competitive firms producing the final consumption good out of labor services and intermediate goods, a continuum of mass $N$ of monopolists producing each a variety $j$ of intermediate goods using the final good as input, and a policy maker.

## 3.1 Final good sector

This sector produces the consumption good $Y_t$ under perfect competition, using a Cobb-Douglas technology that combines labor $L_t$ and a continuum of intermediate inputs $y_t(j)$

whose mass is $N_t$, indexed by $j \in [0, N_t]$ and with associated productivity/quality $A_t(j)$:

$$Y_t = L_t^{1-\alpha} \int_0^{N_t} A_t(j)^{1-\alpha} y_t(j)^{\alpha} dj, \tag{3.1}$$

where $\alpha \in (0, 1)$. Final good firms maximize their profits choosing the optimal quantities of intermediate goods and labor services. The solution to this problem is characterized by the following first order conditions,

$$W_t = (1-\alpha) P_t L_t^{-\alpha} \int_0^{N_t} A_t(j)^{1-\alpha} y_t(j)^{\alpha} dj \tag{3.2}$$

$$p_t(j) = \alpha P_t (L_t)^{1-\alpha} y_t(j)^{\alpha-1} A_t(j)^{1-\alpha}, \tag{3.3}$$

$\forall j \in [0, N_t]$, where $p(j)$ is the price of the intermediate good $j$, $P$ the price of the final good and $W$ the nominal wage.

## 3.2 Intermediate good sector

Following Aghion and Howitt (2008), the intermediate good in sector $j$ is produced by a monopolist using the final good $x_t(j)$, with a linear technology,

$$y_t(j) = x_t(j), \tag{3.4}$$

for $j \in [0, N_t]$. This implies a one to one relationship between intermediate goods, sectors and firms and we will refer to them interchangeably throughout the text. The monopolist chooses price and quantity of the intermediate good to solve

$$\Pi [A_t(j)] \equiv \max_{p_t(j), x_t(j)} p_t(j) y_t(j) - P_t x_t(j)$$

subject to (3.3) and (3.4)

which implies the equilibrium quantity of intermediate good $j$

$$y_t(j) = \alpha^{\frac{2}{1-\alpha}} A_t(j) L_t \tag{3.5}$$

and the equilibrium price of intermediate good $j$

$$p_t(j) = \frac{1}{\alpha} P_t.$$

Accordingly, the equilibrium monopolist's profits in sector $j$ are

$$\Pi\left[A_t\left(j\right)\right] = \psi P_t A_t\left(j\right) L_t \tag{3.6}$$

with $\psi \equiv \left(1-\alpha\right)\alpha^{\frac{1+\alpha}{1-\alpha}}$, the equilibrium aggregate amount of intermediate goods is

$$X_t^y \equiv \int_0^{N_t} y_t\left(j\right) dj = \alpha^{\frac{2}{1-\alpha}} Q_t L_t \tag{3.7}$$

while using (3.5) in (3.1) gives the equilibrium level of final output

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} Q_t L_t.$$

In both equations above, $Q$ represents the aggregate productivity index, defined as

$$Q_t \equiv \int_0^{N_t} A_t(j)\, dj, \tag{3.8}$$

which highlights that, in our economy, aggregate productivity can evolve over time as a result of innovation along either or both of the following margins: *i*) the vertical (intensive) margin, measuring the average quality of existing varieties (i.e., changes in $A$); and *ii*) the horizontal (extensive) margin, measuring the mass of existing varieties (i.e., changes in $N$). We turn now to describing in detail these two margins of innovation.

## 3.3 Innovation

The entrepreneurs running the intermediate-good producing firms belong to a set of potential innovators that compete over who will become the monopolist producer of a given variety $j$. Such potential innovators are indexed according to their relative technical and entrepreneurial skills $x \in [0,1]$, that is drawn from a given unimodal and right-skewed Beta distribution $f(x;\tilde{\alpha},\tilde{\beta})$.

Each period, a potential innovator engages in R&D activity to attempt an innovation that will grant them the status of monopolist in the intermediate-good sector. Once defined the optimal amount of R&D spending necessary to reach the technological frontier, they choose whether to pursue an incremental innovation, improving the quality of an existing variety $j$, or to try and expand the set of varieties producing a radical innovation. In the first case, their probability of success only depends on the level of R&D spending, as in Benigno and Fornaro (2018), among others, while in the second case, it also depends on their idiosyncratic technical and entrepreneurial skills, $x$, in the spirit of Lucas (1978).[7]

#### 3.3.1 Incremental innovation through quality ladders

As in the seminal work of Grossman and Helpman (1991) and Aghion and Howitt (1992), the engine of vertical innovation is underpinned by quality ladders, in the spirit of the Schumpeterian idea of creative destruction (Schumpeter, 1942). We think of this type of innovation as "incremental" in that it yields to improved products but it doesn't have the innovative content to create a brand new fringe.

In each period $t$ and in each sector $j$, an entrepreneur that chooses to attempt an innovation along the vertical margin succeeds with probability $z_t(j)$, that is independent of their technical and entrepreneurial skills $x$. In this case, the innovation creates a new and better version of the intermediate good: the productivity of the variety $j$ improves by a step of size $\gamma$, i.e. $A_t(j) = (1+\gamma) A_{t-1}(j)$. If they fail, the intermediate good remains of the same quality as in $t-1$ and it is produced by another randomly chosen monopolist. Therefore:

$$A_t(j) = \begin{cases} (1+\gamma) A_{t-1}(j) & \text{with prob. } z_t(j) \\ A_{t-1}(j) & \text{with prob. } 1 - z_t(j) \end{cases}$$

In order to generate a successful innovation with probability $z_t(j)$, an entrepreneur must invest in research and development, according to the following production function:

$$z_t(j) = \left[\delta \frac{1+\nu}{\nu} \frac{R_t(j)}{A_t(j)}\right]^{\frac{\nu}{1+\nu}}, \tag{3.9}$$

with $\nu > 0$, and where $R_t(j)$ denotes the amount of final good used in R&D activity. This production function implies that it is more costly to innovate over more advanced goods. The entrepreneur maximizes their expected profits from innovation:

$$d_t(j) \equiv \max_{z_t(j)} z_t(j) \Pi\left[(1+\gamma) A_{t-1}(j)\right] - R_t(j) P_t$$
$$\text{subject to (3.6) and (3.9)}$$

[7]We chose to model incremental innovation as quality ladders (Grossman and Helpman, 1991, Aghion and Howitt, 1992) and radical innovation as varieties expansion (Romer, 1990b) for tractability. These two theories of innovation are well known and widely used in macroeconomic models, especially business cycles ones. Our results are invariant to (i) modeling radical innovation along quality ladders and incremental innovation along varieties expansion and (ii) a more realistic set up in which both types of innovation can occur within the same margin. However in the scenario (ii) the analysis would be much more complicated, shadowing the main punchline of the paper.

leading to the equilibrium innovation intensity in sector $j$

$$z_t(j) = (\delta \psi L_t)^{\nu} = z_t \tag{3.10}$$

for all $j \in [0, N_t]$[8]. Accordingly, the optimal level of R&D spending necessary to achieve the technological frontier is common across varieties and equal to $R_t^* = \frac{\nu}{1+\nu}\delta^{\nu} A_t(\psi L_t)^{1+\nu}$.[9]

The growth rate of sectoral productivity $1 + g_t^A \equiv \frac{A_t}{A_{t-1}}$, therefore satisfies

$$g_t^A = z_t \gamma = \gamma(\delta \psi L_t)^{\nu} \tag{3.11}$$

and the definition of aggregate productivity implies

$$Q_t = N_t A_t. \tag{3.12}$$

Using (3.10), the equilibrium expected profits associated to an incremental innovation are

$$d_t = \frac{z_t}{1+\nu}\psi L_t A_t P_t = \frac{z_t}{1+\nu}\Pi(A_t),$$

which is increasing in employment and thus inherits its cyclicality. This captures the *market size* theory of innovation (Grossman and Helpman, 1991, Romer, 1990b, Aghion and Howitt, 1992), whereby a larger product demand pulls more research effort aimed at improving product quality and hence capturing larger returns.

### 3.3.2 Radical innovation through expanding varieties

The engine of horizontal innovation builds on the expanding variety model of Romer (1990a) and the entrepreneurial-decision model of Lucas (1978). We deem this type of innovation as "radical" because it creates completely new fringes, generates higher expected profits and it requires relatively higher technical and entrepreneurial skills.

In particular, we assume that the number of varieties evolves following

$$N_t = (1 - \Delta) N_{t-1} + N_t^e,$$

where $\Delta$ is an exogenous obsolescence rate, at which varieties vanish, and $N_t^e$ is the cre-

[8]To ensure $z_t \in [0, 1]$ for all $t$, we assume $\delta \leq \psi^{-1}$.

[9]Indeed, equation (3.10) implies that the equilibrium probability of vertically innovating is common across firms. Assuming a uniform initial cross-sectional distribution of sectoral productivities $A_{-1}(j) = A_{-1}$ for all $j \in [0, N_{-1}]$ implies symmetry ex-post: $A_t(j) = A_t$, for all $j$.

ation of new ones, which satisfies the production function

$$N_t^e = n_t N_{t-1}, \tag{3.13}$$

with $n_t$ the mass of innovators that succeed in the expansion of varieties.

To describe the latter, we assume that in each period the prospective innovators can engage in exploration activities. If the activity is successful, innovators become monopolists in charge of the production of the new varieties, and thus they enjoy the profits $\Pi(A_t)$. To try and expand the set of known varieties, entrepreneurs must first invest the same R&D efforts $R^*$ as the vertical innovators, so as to reach the technological frontier $A_t$. Given this level of R&D spending, the probability of success in the exploration activity depends on the idiosyncratic technical and entrepreneurial skills of the potential innovator, according to function $h(x) = \kappa x^{\frac{1}{\varpi}}$ with $\kappa$, $\varpi > 0$ such that $h(x) \in [0,1]$ and $h'(x) > 0$ for all $x \in [0,1]$.

Under perfect competition and free entry among the potential innovators, a prospective entrepreneur with a skill level $x$ optimally chooses to try and innovate along the extensive margin if and only if the implied expected profits are at least as large as those from incremental innovation:

$$h(x)\Pi(A_t) - R_t^* P_t \geq d_t. \tag{3.14}$$

The condition (3.14) thus implies the following threshold on the skill endowment that makes it profitable (in expected terms) to attempt a radical innovation:

$$\underline{x}_t = \underline{x}(L_t) = h^{-1}\Big( (\delta\psi L_t)^{\nu} \Big) = \kappa^{-\varpi} (\delta\psi L_t)^{\nu\varpi}, \tag{3.15}$$

at which $h(\underline{x}_t) = (\delta\psi L_t)^{\nu}$.

This implies that the continuum of prospective innovators is optimally partitioned depending on their technical skills $x$: those endowed with skills $x \geq \underline{x}_t$ optimally choose to try and participate in the creation of new varieties, while those with $x < \underline{x}_t$ optimally limit themselves to incremental innovation.

**Proposition 1.** *At each time $t$, the mass of innovators participating in the creation of new varieties is:*

$$n_t = n(L_t) = \int_{\underline{x}(L_t)}^{1} h(x) f(x)\, dx \tag{3.16}$$

*with $\underline{x}_t$ satisfying (3.15), and is a decreasing function of aggregate employment: $\frac{\partial n_t}{\partial L_t} < 0$.*

**Proof.** To show that the mass of innovators participating in the creation of new varieties is decreasing in employment, we compute the derivative of $n_t$ with respect to $L_t$. Using the Leibniz integral rule we obtain:

$$\frac{\partial n_t}{\partial L_t} = -h(\underline{x}_t) f(\underline{x}_t) \frac{\partial \underline{x}_t}{\partial L_t}. \tag{3.17}$$

Note now that the derivative of the threshold is:

$$\frac{\partial \underline{x}_t}{\partial L_t} = \nu \varpi \kappa^{-\varpi} \frac{(\delta \psi L_t)^{\nu \varpi}}{L_t} > 0. \tag{3.18}$$

Using $h(\underline{x}_t) = (\delta \psi L_t)^{\nu}$, as well as (3.10), into (3.17), we finally obtain:

$$\frac{\partial n_t}{\partial L_t} = -\nu \varpi f(\underline{x}_t) \frac{z_t^{1+\varpi}}{\kappa^{\varpi} L_t} < 0.$$

■

Intuitively, a fall in aggregate employment reduces the equilibrium probability of success of an attempt at an incremental innovation, as implied by equation (3.10), and thus the expected profits of exploitative activity. Thereby, this raises the incentive to try and expand the set of varieties, by enlarging the set of technical skills that can be exploited to increase the probability of success in the horizontal dimension, and the associated expected profits. This is consistent with the *opportunity cost* theory of innovation (Aghion and Saint-Paul, 1991, Aghion and Saint-Paul, 1998b, Gali and Hammour, 1992, Cooper and Haltiwanger, 1993, Caballero and Hammour, 1994, Hall, 1991, Saint-Paul, 1997, De-Long, 1990, Canton and Uhlig, 1999 and King and Robson, 1989) and captures the idea that, during downturns, the foregone expected profits of incrementing the quality of existing products are lower, and hence it becomes more appealing to engage in exploration research that gives access to higher expected returns.

Given this specification, the rate of growth of varieties is

$$1 + g_t^N \equiv \frac{N_t}{N_{t-1}} = 1 - \Delta + n(L_t), \tag{3.19}$$

which is decreasing in employment, as it becomes harder to create new products when the economy is booming and expected returns from incremental innovation are higher.

### 3.3.3 The U-shaped growth equation

The previous two sections clarify that the rate of growth of aggregate productivity will be the result of the interaction of the two driving forces in this economy: i) incremental innovation through quality ladders, captured by (3.11), and ii) radical innovation through expansion of varieties, captured by (3.19). From the definition (3.12), it follows that the equilibrium rate of growth in aggregate productivity satisfies

$$1 + g_t \equiv \frac{Q_t}{Q_{t-1}} = (1 + g_t^N)(1 + g_t^A) = \Big(1 + \gamma(\delta\psi L_t)^\nu\Big)\Big(1 - \Delta + n(L_t)\Big). \tag{3.20}$$

The above equation, together with Proposition 1, shows that the interaction of the counter-cyclical mechanism on the horizontal margin of innovation with the pro-cyclical one of vertical innovation may result in a non-monotonic relationship between aggregate productivity and employment, depending on parameter values. On the one hand, for high levels of employment the vertical margin of innovation is dominant, and aggregate productivity growth is thus increasing in $L$. On the other hand, in deep recessions bringing the employment level sufficiently low, the horizontal margin of innovation kicks in, and aggregate productivity growth becomes decreasing in $L$.

In an appropriate subset of the parameter space, the relation (3.20) provides the U-shaped growth equation that is consistent with the empirical evidence discussed in Section 2. As we will show, this non-monotonic behavior is key, as it introduces an additional channel through which the economy can end up in an adverse steady state. This in turn has important implications in the analysis of recessions and the industrial policies the government may undertake to steer the economy out of an unemployment steady state.

We will refer to equation (3.20) as $GG$ throughout the rest of the paper.

## 3.4 Households

This economy is populated by a unit mass of households deriving utility from consumption of the final good. Their lifetime utility is

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} \right) \right],$$

where $C_t$ denotes consumption in units of the final good, $\beta \in (0,1)$ the subjective discount factor and $\sigma > 1$ is the relative risk aversion coefficient (and the the inverse of the elasticity of intertemporal substitution). Within each household there is a unit measure of members that are willing to supply labor services in the labor market. Within each household there is perfect consumption sharing.

As in Benigno and Fornaro (2018), households face idiosyncratic unemployment risk: at the beginning of each period, each household will be unemployed with constant probability $p$. If the household is unemployed, all members of the household are out of a job, and the household receives an unemployment subsidy that makes its income equal to $b$ times the income of the employed household, with $b < 1$. If the household is employed, on the other hand, its members supply inelastically labor services on the labor market. Because of nominal rigidities, however, the equilibrium may imply a labor demand from firms that is insufficient to absorb all labor supply and a measure $1 - L_t \in (0, 1)$ of the household members may be unemployed. The unemployed members of the employed household, however, will enjoy the same level of consumption of the employed ones, due to the perfect risk-sharing mechanism at work within the household.[10]

Households have access to risk free bonds $B_t$ that pay the nominal interest rate $i_t$ and own all the firms, from which they receive dividends $D_t$.

Thus, the flow budged constraint reads

$$P_t C_t + \frac{B_{t+1}}{1+i_t} = \mathbb{I} W_t L_t + B_t + D_t + T_t$$

where the indicator function $\mathbb{I}$ takes value 1 if the household is employed and 0 otherwise, while the transfer $T_t$ will be a subsidy for the unemployed and a tax for the employed.[11]

The household maximizes her expected lifetime utility subject to the flow budget constraint, a no-borrowing constraint for unemployed households, $B_{t+1} \geq 0$, no trade in firms' shares and a standard no-Ponzi game condition. Noticing that the borrowing constraint binds only for unemployed households, this problem leads to the following first order conditions:

$$\mu_t = \frac{\left(C_t^e\right)^{\sigma}}{P_t} \tag{3.21}$$

$$\mu_t = \beta\left(1+i_t\right)\mathbb{E}_t \mu_{t+1} \tag{3.22}$$

together with the standard transversality condition $\lim_{T\to\infty} \mathbb{E}_t \beta^T \frac{Uc,T}{Uc,t} B_T = 0$, where $C_t^e$ denotes consumption of employed households, and by the assumption on unemployed households' income, $C_t^{ne} = bC_t^e < C_t^e$.

[10] Given this structure, there are two types of unemployment. The first type is "exogenous", of mass $p$. The second type is instead "endogenous", as it is generated by the equilibrium labor demand, and it is measured by the mass $(1-p)(1-L_t)$. Hence, households accounting is summarised as follows:

$$1 = \underbrace{p}_{\text{exogenous unemployed}} + \underbrace{(1-p)(1-L_t)}_{\text{endogenous unemployed}} + \underbrace{(1-p)L_t}_{\text{employed}}.$$

[11] In particular, for the employed $T_t = -\frac{p}{1-p}\frac{bW_tL_t+(b-1)N_td_t}{1+\frac{bp}{1-p}}$ and for the unemployed $T_t = \frac{bW_tL_t+(b-1)N_td_t}{1+\frac{bp}{1-p}}$.

### 3.5 Nominal rigidities and monetary policy

We introduce nominal frictions so that there will be a role for monetary policy in this framework and involuntary unemployment will be possible. Following Benigno and Fornaro (2018), we start by assuming that wages evolve according to constant inflation,

$$W_t = \bar{\pi}^w W_{t-1} \tag{3.23}$$

and will relax later this assumption introducing a wage Phillips curve.

Monetary policy follows an interest rate rule considering the ZLB (i.e. $i_t \geq 0$),

$$1 + i_t = \max\left\{(1+\bar{\imath})\, L_t^{\phi}, 1\right\} \tag{3.24}$$

where $\phi > 0$ and $\bar{\imath} \geq 0$. Under this specification and combining (3.2) and (3.5), prices are

$$P_t = \omega^{-1} \frac{W_t}{Q_t} \tag{3.25}$$

where $\omega^{-1} \equiv \frac{1}{1-\alpha}\alpha^{\frac{2\alpha}{\alpha-1}}$ while stationary real wages are $\frac{W_t}{P_t Q_t} \equiv \omega$. Combining the equation for prices (3.25) and the law of motion of wages (3.23) one gets the price inflation

$$\pi_t \equiv \frac{P_t}{P_{t-1}} = \frac{\bar{\pi}^w}{1+g_t}. \tag{3.26}$$

### 3.6 Equilibrium

The resource constraint implies that the final output is used either to consume, to produce the intermediate good or employed in research activities for innovation:

$$Y_t = C_t + \int_0^{N_t} y_t(j)\, dj + \int_0^{N_t} R_t(j)\, dj.$$

Using (3.5), (3.8), (3.9) and (3.10) one gets

$$C_t = \Psi L_t Q_t - \frac{\nu \delta^{\nu}}{1+\nu} (\psi L_t)^{1+\nu} Q_t,$$

where $\Psi \equiv \alpha^{\frac{2\alpha}{1-\alpha}}\left(1-\alpha^2\right)$. Defining the stationary level of consumption as $c_t = {}^{C_t}/_{Q_t}$ the resource constraint can be written as

$$c_t = \Psi L_t - \frac{\nu \delta^{\nu}}{1+\nu} (\psi L_t)^{1+\nu}. \tag{3.27}$$

Turning to households, combining (3.21), (3.22) and (3.26), and using the fact that $C_t = pC_t^{ne} + (1-p)\,C_t^e$ and that by the assumption on unemployed households' income $C_t^{ne} = bC_t^e < C_t^e$, the Euler equation is

$$\bar{\pi}^w c_t^{-\sigma} = \beta\,(1+i_t)\,\rho\mathbb{E}_t\left\{c_{t+1}^{-\sigma}\,(1+g_{t+1})^{1-\sigma}\right\} \tag{3.28}$$

where $\rho = 1 + p(b^{-\sigma} - 1) > 1$, and we assume $\sigma > 1$ so that the income effect dominates and there is a positive relationship between current consumption and future expected productivity.

This analysis leads to the following definition:

**Definition 1.** *A stationary equilibrium in this economy is a sequence* $\{c_t, g_t, L_t, i_t, n_t, \underline{x}_t\}_{t=t_0}^{\infty}$ *that satisfies (3.15), (3.16), (3.20), (3.24) (3.27) and (3.28), and* $L_t \leq 1$ *for each* $t \geq t_0$*, given the parameters* $\sigma, \bar{\pi}^w, \rho, \beta, \Psi, \delta, \gamma, \Delta, \psi, \kappa, \varpi, \nu, \bar{\imath}, \phi$*, a function* $h(x)$ *with* $h(x) \in [0,1]$ *and* $h'(x) > 0$*, and a density function* $f(x)$*.*

## 3.7 Steady State Analysis

Turning off aggregate uncertainty and focusing on constant values for consumption $c$, labour $L$, aggregate productivity growth $g$ and for the nominal interest rate $i$, equilibria are characterized by the solutions to the following system of equations:

$$(1+g)^{\sigma-1} = \frac{\rho\beta\,(1+i)}{\bar{\pi}^w} \tag{3.29}$$

$$c = \Psi L - \frac{\nu\delta^\nu}{1+\nu}\,(\psi L)^{1+\nu} \tag{3.30}$$

$$1+g = \Big(1+\gamma(\delta\psi L)^\nu\Big)\Big(1-\Delta+n(L)\Big) \tag{3.31}$$

$$n(L) = \int_{x \geq \underline{x}(L)} h(x) f(x)\,dx \tag{3.32}$$

$$\underline{x}(L) = h^{-1}\Big(\,(\delta\psi L)^\nu\,\Big) \tag{3.33}$$

$$1+i = \max\left[(1+\bar{\imath})\,L^\phi, 1\right]. \tag{3.34}$$

The system (3.29)-(3.34), can be described by two relevant equations. Combining the Euler equation (3.29) with the monetary policy rule (3.34) one gets an aggregate-demand type of relationship between productivity growth and labour:

$$1+g = \max\left\{\left(\frac{\rho\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}, \left[\frac{\rho\beta}{\bar{\pi}^w}\,(1+\bar{\imath})\,L^\phi\right]^{\frac{1}{\sigma-1}}\right\} \tag{3.35}$$

and it is easy to see that when $i > 0$ the second element in the max dominates and the relationship between labour $L$ and productivity growth $g$ is positive; when instead $i = 0$ the relationship between labour $L$ and aggregate productivity $g$ becomes flat because the central bank is not anymore able to respond to unemployment decreasing further the policy rate.[12] We will refer to equation (3.35) as $AD$. The other relevant equation is the $GG$ evaluated at the steady state (equation 3.31).

The system (3.29)-(3.34) gives rise to multiple equilibria, under some conditions. In particular a full employment steady state emerges naturally and with few assumptions while multiple unemployment steady states can emerge if radical innovation is not relatively strong enough.

### 3.7.1 Full employment Steady State

A full employment steady state is characterized by $L^F = 1$. From (3.31) we get

$$1 + g^F = \left(1 + \gamma(\delta\psi)^\nu\right)\left(1 - \Delta + n(1)\right) \tag{3.36}$$

(3.33) implies $\underline{x}^F = \kappa^{-\varpi}\left(\delta\psi\right)^{\nu\varpi}$ while (3.29) yields

$$1 + i^F = \left(1 + g^F\right)^{\sigma-1}\frac{\bar{\pi}^w}{\rho\beta}$$

and monetary policy supports the full employment steady state by setting $\bar{\imath} = i^F$. Lastly, from the resource constraint (3.30) we can recover the consumption level

$$c = \Psi - \frac{\nu\delta^\nu}{1+\nu}\psi^{1+\nu}.$$

**Assumption 1.** *The parameters satisfy:*

$$\left(1 + \gamma(\delta\psi)^\nu\right)\left(1 - \Delta + n(1)\right) > 1 \tag{3.37}$$

$$\delta\psi \leq 1 \tag{3.38}$$

$$\left[\left(1 + \gamma(\delta\psi)^\nu\right)\left(1 - \Delta + n(1)\right)\right]^{\sigma-1}\frac{\bar{\pi}^w}{\rho\beta} > 1 \tag{3.39}$$

$$\Phi > 0 \tag{3.40}$$

$$\phi > (\sigma - 1)\,\Phi. \tag{3.41}$$

[12] Note that $i > 0$ occurs when $L > \bar{L} \equiv \frac{\bar{\pi}^w}{\rho\beta}\frac{1}{(1+\bar{\imath})}$. while $i = 0$ when $L \leq \bar{L}$.

*where* $\Phi \equiv \left[\frac{\gamma\nu(\delta\psi)^{\nu}}{1+\gamma(\delta\psi)^{\nu}} + \frac{n'(1)}{1-\Delta+n(1)}\right]$.

**Proposition 2.** *(Existence, uniqueness and local determinacy of the full employment steady state) Suppose Assumption 1 is satisfied. Then there exists a unique full employment steady state characterised by* $L^F = 1$, $g^F > 0$, $i^F > 0$ *and* $c^F > 0$ *and it is locally determinate.*

**Proof.** Please refer to Appendix A.1. ∎

Assumption (3.37) ensures that the full-employment growth rate is positive, while assumption (3.38) makes sure that the full-employment consumption level is positive and that the innovation probability lies between zero and one, i.e. $z_t \in [0,1]$. Analogously, assumption (3.39) guarantees that the interest-rate target is positive and consistent with the full-employment steady state. Finally, assumption (3.40) implies that the *GG* is locally increasing around the full-employment steady state, and assumption (3.41) ensures that the central bank is sufficiently responsive to fluctuations in employment to rule out sunspot fluctuations that may keep the equilibrium away from the full-employment steady state. Note that assumption (3.41) effectively requires that—in a neighborhood of the full-employment steady state—the slope of the *AD* schedule, i.e. $\phi/(\sigma-1)$, be larger than the slope of the growth equation, i.e. $\Phi$. This interpretation clarifies that the condition for local determinacy is milder with respect to the one in Benigno and Fornaro (2018), because the horizontal margin of innovation tends to reduce the slope of the growth equation for any level of employment, thus reducing the threshold level of policy responsiveness required to implement determinacy.

### 3.7.2 Unemployment Steady States

An unemployment steady state is characterised by $i = 0$ which, by equation (3.29) implies

$$g^U = \left(\frac{\rho\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} - 1 < g^F, \tag{3.42}$$

where the inequality is implied by assumption (3.39).

Now note that in our framework, due to the U-shape of the *GG* equation, neither the existence of an unemployment steady state nor its uniqueness are ensured by the assumptions made so far. In particular, existence of at least one unemployment steady state requires that the minimum of the *GG* (with respect to *L*) be less than, or at most equal to, the level of growth at the zero lower bound consistent with the horizontal portion of the *AD*, $g^U$, and that the slope of the *GG* at $L = 1$ be smaller than the one of the AD. This latter condition is ensured by assumption (3.41), as discussed above, while the former depends on the shape of the function governing the radical margin of innovation. We can characterize this result in the following proposition:

**Proposition 3.** *(Existence, multiplicity and local determinacy of stagnation traps)*
*Suppose assumption 1 is satisfied, and that, in addition, the parameters satisfy*

$$\nu \leq \frac{1+\alpha}{\alpha} \tag{3.43}$$

$$\rho\beta > \bar{\pi}^w \tag{3.44}$$

$$n(0) - \Delta \geq g^U. \tag{3.45}$$

*Then, the following holds:*

1. *Existence and multiplicity*
   - *at least one unemployment steady state exists if and only if the following is satisfied:*

$$\min_L \left(1+\gamma(\delta\psi L)^\nu\right)\left(1-\Delta+n(L)\right) \leq 1+g^U \tag{3.46}$$

   - *if condition (3.46) holds with equality, there is exactly one stagnation trap*
   - *otherwise, there are exactly two stagnation traps*

   *in either case, these unemployment steady states satisfy*

$$i^U = 0, \qquad g_1^U = g_2^U = g^U < g^F, \qquad 0 < c_1^U \leq c_2^U < c^F, \qquad 0 < L_1^U \leq L_2^U < L^F = 1.$$

2. *Local determinacy*
   - *when there is only one stagnation trap, it is locally indeterminate*
   - *when there are two stagnation traps, the one associated with the lower employment rate, $L_1^U$, is locally determinate while the other, $L_2^U$, is locally indeterminate.*

**Proof.** Please refer Appendix A.2. ■

Assumption (3.41) ensures that the upward sloped portion of the $AD$ lies below the $GG$ over a left neighbourhood of $L = 1$ while assumption (3.46) guarantees that the $GG$ schedule has a minimum below or on the horizontal portion of the $AD$ so that at least one unemployment steady state exists. It also implies that, if two unemployment steady states exist—that is if (3.46) holds with strict inequality—then one is associated to a locally downward-sloping $GG$ and the other with an upward-sloping $GG$. Assumption (3.44) ensures productivity growth to be positive in the unemployment steady states. Assumption (3.38) instead ensures that consumption is positive for each and any $L_i^U$ with $i = 1, 2$, and (3.43) that consumption is increasing in employment for any $L \in [0, 1]$, implying $0 < c_1^U \leq c_2^U < c^F$.

The blue and red solid lines in Figure 1 display the relevant $AD$ and $GG$ schedules in our economy, and the multiple unemployment steady states they can support.

To understand the nature and implications of this multiplicity, consider as a benchmark the simplified case—nested in our environment—where innovation can only occur through the vertical margin, as in Benigno and Fornaro (2018), i.e. $\Delta = \kappa = 0$, implying $n(L) = 0$ for any $L \in [0, 1]$ and $g^N = 0$. The $GG$ schedule in this case is monotonically increasing and the market size effect is the only driver of innovation:

$$1 + g = 1 + g^A = 1 + \gamma(\psi\delta L)^\nu.$$

This case is displayed by the dashed red line in Figure 1. In an appropriate subset of the parameter space, two steady states exist: the full employment equilibrium, $(1, g^F)$, and the stagnation-trap equilibrium, $(L^U, g^U)$, with $L^U < 1$ and $g^U < g^F$.[13] In particular, once condition (3.44) is satisfied, the stagnation-trap equilibrium always exists. In this unemployment steady state, monetary policy is constrained by the zero lower bound, and it is thus unable to grant the necessary responsiveness needed to rule out sunspot fluctuations for any level of employment. This implies (local) indeterminacy of the stagnation trap in economies where the only margin of innovation is the incremental one, where the $GG$ schedule is monotonic increasing in employment.

Compared to this benchmark, the general specification of our model, including heterogeneous innovation, has several implications.

We start by noting that the engine of radical innovation is in principle able to prevent stagnation traps altogether. Indeed, whether or not assumption (3.46) is satisfied depends on the particular shape of radical innovation, and in particular on the parameterization of function $h(x)$. The easier it is to create a new variety, the higher the relative strength of the horizontal margin of innovation over the vertical one for any given level of employment, and *a fortiori* during recessions. This suggests that there exists a subset of the parameter space of function $h(x)$ for which assumption (3.46) is violated, the $GG$ schedule lies entirely above the $1 + g^U$ line, and stagnation traps do not arise at all. This case is displayed

[13] For the sake of the comparative discussion, we are assuming—without loss of generality—that the full-employment steady states in the two specifications of the model—with and without horizontal innovation—are the same, which requires setting $\Delta = n(1)$ in the general model.

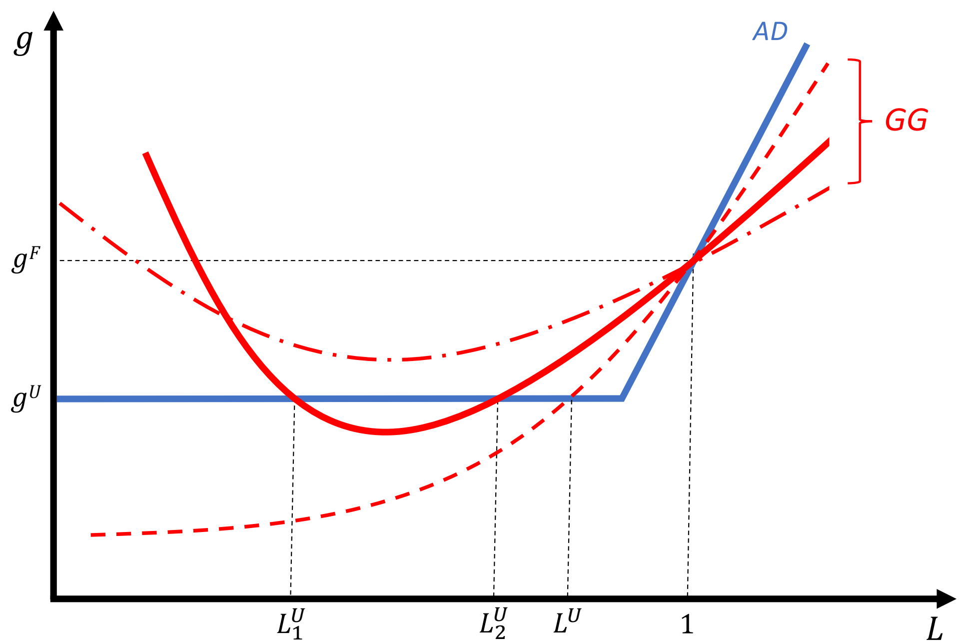


**Figure 1:** Multiple stagnation traps.

by the dashed-dotted red line in Figure 1. This also means that any policy intervention that affects the minimum threshold ability $\underline{x}_t$ needed to create a new variety and makes it easier, is in principle able to prevent the formation of stagnation traps or to allow the economy to escape from them, as we discuss in Section 4.

However, while radical innovation may rule out stagnation traps, when such traps do arise they are generally harsher than in the simplified case, in two respects. The first is related to the fact that the *GG* schedule is flatter around the full employment steady state, as discussed. This on the one hand reduces the burden on monetary policy to induce determinacy of the full-employment equilibrium, through an aggressive responsiveness, but on the other hand it implies that—if the full-employment equilibrium is the same—the stagnation traps in the general model are associated to a lower level of employment compared to the simplified one, i.e. $L_1^U \leq L_2^U < L^U$. This is related to the fact that, in the general model, falling employment has a weaker effect on productivity growth because of the counter-cyclical radical margin of innovation, thereby requiring a larger fall in employment to reach the demand-determined equilibrium rate of growth $g^U$.

The second respect in which stagnation traps are harsher when heterogeneous innovation is accounted for is related to the U-shape of the *GG* schedule. As stated in Proposition 3, if condition (3.46) is satisfied, a second unemployment steady-state exists, which is worse than the first one for two reasons: $i)$ it is associated to a lower employment rate, i.e. $L_1^U < L_2^U$, and, most importantly, $ii)$ it is locally determinate. In our economy, indeed, the lack of an endogenous policy response at the ZLB is not necessarily associated to an indeterminate equilibrium. Since equilibrium radical innovation is counter-cyclical, this margin is in principle able to provide a shield against sunspot fluctuations—preventing downward revisions in employment expectations to become self-fulfilling—if it is strong enough. This in particular occurs when the unemployment steady state is associated to the downward-sloping part of the $GG$ schedule.

Interestingly, depending on the calibration of the model, the two unemployment equilibria could almost overlap. In such a scenario, one would focus only on the stable unemployment equilibrium, as any small structural shock in its neighborhood would inevitably yield to its basin of attraction. This is a very important feature of our framework: *stagnation traps can indeed be stable and absorbing states*, and structural shocks can pull the economy towards their gravitational basin. This feature can help rationalize the *fundamental* nature of persistent slumps and liquidity traps, such as the ones experienced in the Euro Area and Japan.

### 3.8 A numerical illustration

In this section we provide a numerical illustration of the implications of heterogeneous innovation for the unemployment equilibria, in a calibrated version of the model extended to allow for a more meaningful form of nominal rigidities. In particular, we introduce downward nominal-wage rigidity as in Benigno and Fornaro (2018), which gives rise to a wage Phillips curve:

$$\frac{W_t}{W_{t-1}} = \pi_t^w \geq \theta\left(L_t\right)$$

with $\theta' > 0$ and $\theta\left(1\right) = \bar{\pi}^w$. For $\pi_t^w \geq \bar{\pi}^w$ output is at potential, otherwise there is a positive relationship between inflation and the output gap.

Now the central bank responds to deviations of wage inflation from a target $\pi^*$, still following a truncated interest rate rule:

$$1 + i_t = \max\left[\left(1+\bar{\imath}\right)\left(\frac{\pi_t^w}{\pi^*}\right)^{\phi}, 1\right] \tag{3.47}$$

where $\pi^* \geq \bar{\pi}^w$ so that when the central bank hits the target the economy is at full employment. Moreover, we assume the interest rate target to be $1 + \bar{\imath} = \frac{\pi^*}{\rho\beta}\left(1+g^F\right)^{\sigma-1}$, where $g^F$ is the full-employment rate of growth in productivity.[14]

In this set up, a steady state is characterized by the solution to the system described by (3.31), (3.47) evaluated at the steady state, (3.30) and

$$(1+g)^{\sigma-1} = \frac{\rho\beta(1+i)}{\pi^w} \tag{3.48}$$

$$\pi^w \geq \theta(L). \tag{3.49}$$

Combining (3.48), (3.47) and (3.49), the new AD is

$$1+g = \max\left\{\left(\frac{\rho\beta}{\theta(L)}\right)^{\frac{1}{\sigma-1}}, \left[\frac{\rho\beta}{\theta(L)}(1+\bar{\imath})\left(\frac{\theta(L)}{\pi^*}\right)^{\phi}\right]^{\frac{1}{\sigma-1}}\right\} \tag{3.50}$$

which is characterized by a negative relationship between productivity growth and employment for values of labour $L$ low enough so that the zero lower bound binds.

**Proposition 4.** *(Local determinacy of the full employment steady state) Suppose Assumption 1 is satisfied, where condition (3.41) is modified as follows:*

$$\vartheta\phi > (\sigma-1)\Phi + \vartheta, \tag{3.51}$$

*with $\vartheta > 0$ governing the slope of the Phillips curve:*

$$\theta(L) \equiv \pi^* L^{\vartheta}. \tag{3.52}$$

*Then the full employment steady state characterised by $L^F = 1$, $g^F > 0$, $i^F > 0$ and $c^F > 0$ is locally determinate.*

**Proof.** Please refer to Appendix A.3. ■

The above proposition, and in particular condition (3.51), emphasizes that, in order for the full-employment steady state to be locally determinate, monetary policy must be relatively more responsive to employment than in the simple model, in order to counteract the inflationary implications of a positively-sloped Phillips curve. To see this, note first that the policy rule in the general model responds to deviations of wage inflation from target, while in the simple model it responds directly to deviations of employment from target. Nonetheless, the specific functional form assumed for the Phillips curve—i.e. equation (3.52)—implies that we can re-cast equation (3.47) in a form analogous to (3.24),

[14] This guarantees that when $\pi^w = \pi^*$ then by (3.47) $i = \bar{\imath} \implies g = g^F$ and by (3.31) $L = 1$.

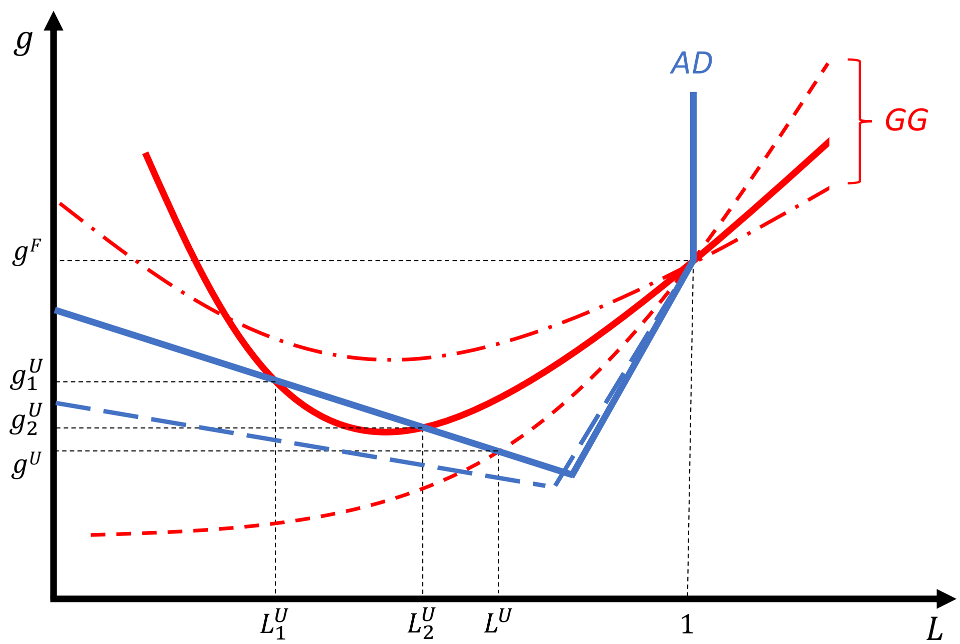


**Figure 2:** Existence and multiplicity of stagnation traps with the Phillips curve. Solid blue line: aggregate demand with high wage flexibility. Dashed blue line: aggregate demand with low wage flexibility.

to highlight the degree of responsiveness to employment:

$$1 + i_t = \max\left[(1+\bar{\imath})\, L_t^{\vartheta\phi}, 1\right],$$

where the implied response coefficient to employment is $\vartheta\phi$. Moreover, Proposition 4 also implies that local determinacy of the full-employment steady state requires this response to be strong enough to make the $AD$ schedule steeper than the $GG$ (see Appendix A.3).

Figure 2 displays the set of equilibria in this extended model, for the same alternative specifications of the growth equation we used in Figure 1. As in the simple case, when the only margin of innovation is the vertical one, and the growth equation is thereby monotonic upward-sloping, the existence of a stagnation trap is granted, and the associated unemployment steady state $\left(L^U, g^U\right)$ is indeterminate. In addition—as argued in Benigno and Fornaro (2018)—higher wage flexibility leads to better outcomes in terms of productivity growth and employment in the stagnation trap. Figure 2 shows that accounting for heterogeneous innovation affects all these features.

**Proposition 5.** *(Local determinacy of the unemployment steady states) Suppose the assumptions of Proposition 3 are satisfied, that the Phillips curve is described by* (3.52)*, and that at least one* *unemployment steady state exists, satisfying*

$$i^U = 0, \qquad g_1^U \geq g_2^U < g^F, \qquad 0 < L_1^U \leq L_2^U < L^F = 1.$$

*Then each unemployment steady state is locally determinate if and only if the local slope of the AD schedule around that steady state is higher than the one of the GG schedule, which requires:*

$$\Phi_0 < \frac{-\vartheta}{\sigma - 1} < 0, \tag{3.53}$$

*where* $\Phi_0$ *is the slope of the GG locus and* $\frac{-\vartheta}{\sigma-1} < 0$ *the slope of the* $AD$ *locus in a liquidity trap.*

*As a consequence, if there is only one unemployment steady state, it is locally indeterminate. Otherwise, if there are two unemployment steady states, the one associated with the lower employment rate,* $L_1^U$*, is locally determinate while the other,* $L_2^U$*, is locally indeterminate.*

*Unlike in the simple model, the locally indeterminate equilibrium does not need to lie on the upward-sloping part of the GG schedule.*

**Proof.** Please refer to Appendix A.4. ■

First, as in the simple model, existence of stagnation traps is not granted in spite of the Phillips curve if the engine of horizontal innovation is strong enough (dashed-dotted line in the figure). Moreover, when stagnation traps do exist, they are in general characterized by multiple unemployment steady states. Of these multiple steady states, the one associated with the lower equilibrium unemployment $\left(L_2^U, g_2^U\right)$ is locally indeterminate, while the one associated with the higher unemployment $\left(L_1^U, g_1^U\right)$ is locally determinate, as in the simple model. An interesting additional implication compared to the simple model is that the determinate stagnation trap is not only associated with a higher unemployment, but also with a higher growth rate, compared to the indeterminate trap. This allows the model to rationalize the existence of "jobless-growth traps", where the growth rate is high, but the employment rate is low. To emphasize this difference, henceforth we are going to refer to the indeterminate equilibrium as "stagnation trap", and use the term "jobless-growth trap" for the determinate one.

Second, the role of wage flexibility is now more effective than in the case of a monotonic growth equation. Note indeed that a lower wage flexibility is associated with a flatter downward-sloping part of the AD schedule. On the one hand, this implies that the stagnation and the jobless-growth traps become relatively more similar in terms of both unemployment and growth rates, as the intersection points get closer to each other. There exists in particular a specific degree of wage stickiness such that the downward-sloping part of the $AD$ is tangent to the $GG$ equation, whereby the two equilibria overlap in a sin-

**Table 3:** Calibrated Parameters

| Parameter | Description | Value | Source or Target |
|---|---|---|---|
| $1/\sigma$ | Elasticity of int. substitution | 0.5 | Benigno and Fornaro (2018) |
| $\beta$ | Discount factor | 0.96 | Benigno and Fornaro (2018) |
| $\rho$ | Idiosyncratic risk | 1.092 | full-employment real interest rate = 1.5% |
| $\pi^*$ | Wage inflation target | 1.052 | full-employment price inflation = 2% |
| $1-\alpha$ | Share of labour in gross output | 0.930 | full-employment R&D-to-GDP = 2% |
| $\gamma$ | Incremental innovation step size | 1.230 | Match the data |
| $\nu$ | Incremental innovation elasticity to R&D parameter | 1.346 | Match the data |
| $\delta$ | Incremental innovation scale parameter | 14.12 | Match the data |
| $\Delta$ | Obsolescence rate of varieties | 0.375 | Match the data |
| $\kappa$ | Radical innovation scale parameter | 0.535 | Match the data |
| $\varpi$ | Radical innovation exponential parameter | 7.274 | Match the data |
| $\tilde{\alpha}$ | Beta Distribution shape parameter | 0.981 | Match the data |
| $\tilde{\beta}$ | Beta Distribution shape parameter | 5.499 | Match the data |
| $\vartheta$ | Phillips curve parameter | 0.34 | output gap in stagnation traps $\in$ [5%, 10%] |

gle stagnation trap. As in the simple model, in this case the unemployment equilibrium is indeterminate. On the other hand, this also implies that if wages are sticky enough, the existence of unemployment steady states may be ruled out altogether, with the only remaining steady state being the full-employment equilibrium.

Within this extended framework, we can provide a numerical illustration of the implications of the U-shaped growth equation for the economy.

To calibrate the structural parameters, we adopt the following simple strategy. Whenever possible, we rely on convention or related literature. When it comes to the parameters shaping the growth equation—along both the incremental and radical margins—we optimize their calibration in order to fit the quadratic relation between growth and employment implied by the estimated parameters of the regression model (2.5), over the range of employment rates that includes our sample. Moreover, when it comes to the Phillips curve, we calibrate the parameter governing the slope, $\vartheta$, in order for the two unemployment steady states to imply levels of the output gap between 5 and 10 percent, in line with the observed slumps in the post-GFC period in the US and the Euro Area. Table 3 collects the full set of calibrated values.

This calibrated example implies the two schedules plotted in Figure 3, where, in order to plot the upward-sloping part of the $AD$ schedule, we further calibrate the response coefficient of the Taylor Rule to $\phi = 3.5$, which implies an overall response to employment

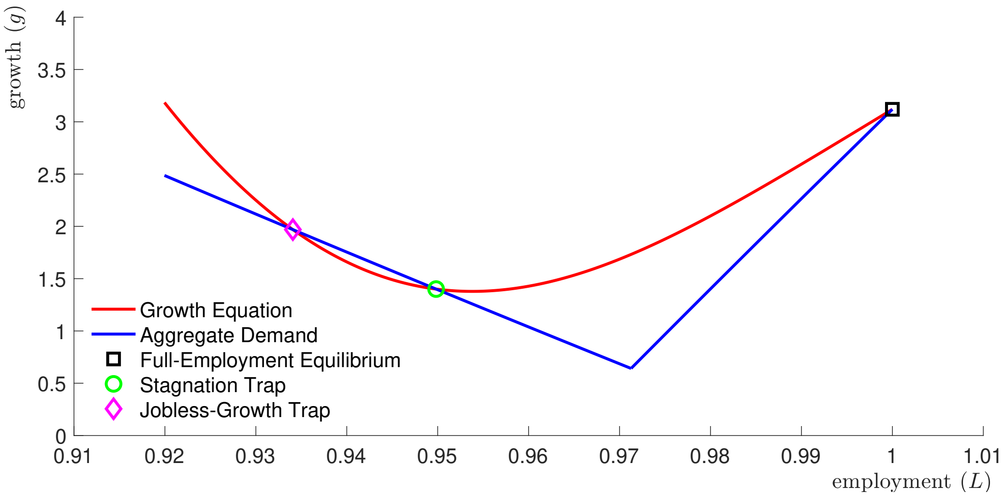


**Figure 3:** Equilibria in the calibrated example. The $y-$axis measures the growth rate $g$, in percentage points.

of $\vartheta\phi = 1.2$.[15] We then report in Table 4 the implied value for a set of variables of interest, to compare the three steady state that arise in this calibrated example.

As the figure shows, the economy in this numericall illustration features three possibile equilibria: a full-employment steady state, a locally indeterminate stagnation trap with positive unemployment and low growth, and a locally determinate jobless-growth trap with higher unemployment than the stagnation trap but also higher growth.

Table 4 displays the implications of the several equilibria for a set of variable of interest. The first column shows the case of the full-emloyment equilibrium, where the 3.16% growth rate is the one implied at full employment by the growth equation estimated using the empirical model (2.5). The targeted levels of 1.5% and 2% respectively for the real interest rate and price inflation imply a nominal interest rate of about 3.5% and a wage inflation of about 5.2%, while the output gap is zero by definition of full employment.

The second column shows the case of the stagnation trap, where the nominal interest rate is at the zero-lower bound, and employment falls by 5 percentage points. This equilibrium is characterized by a strong fall in both productivity growth— which is less than a half of the full-employment level—and the real interest rate—which turns strongly negative. As a consequence, this equilibrium features positive price inflation (close to the target) and a lower wage inflation, about two thirds of the full-employment level.

The third column shows the jobless-growth trap, where on the one hand employment falls by an additional 1.6 percentage points relative to the stagnation trap, while on the other hand productivity growth falls much less, to about two thirds of the full-

[15]Note that this response coefficient does not affect any of the steady-state equilibria. Therefore, we choose an arbitrary value—such that it satisfies the condition for local determinacy of the full-employment equilibrium—for the sole purpose of the plot.

**Table 4:** Numerical Illustration

| | Full-employment steady state | Unemployment steady states | |
|---|---|---|---|
| | | Stagnation Trap | Jobless-Growth Trap |
| Productivity growth | 3.16 | 1.41 | 1.99 |
| Output gap | 0.00 | 5.00 | 6.59 |
| Nominal interest rate | 3.53 | 0.00 | 0.00 |
| Real interest rate | 1.50 | −1.95 | −0.81 |
| Price inflation | 2.00 | 1.97 | 0.81 |
| Wage inflation | 5.23 | 3.41 | 2.82 |

employment benchmark. This equilibrium features very realistic implications also for the real interest rate and inflation rates, with the real rate turning slightly negative, price inflation being moderately positive but less than half its target level, and wage inflation falling to about half the full-employment benchmark.

# 4 Stagnation Traps and Economic Policy

This section discusses some implications for policy that can be drawn from the analysis of the previous sections, focusing in particular on growth policies.[16]

Assume the policy maker introduces a constant subsidy $s \in (0,1)$ on radical innovation, that leads to the creation of new varieties. The subsidy is financed entirely with a lump-sum tax on households.[17] The free entry condition in the (3.14) becomes

$$h(x)\Pi\left(A_t\right) - (1-s)R_t^* P_t \geq d_t. \tag{4.1}$$

As a consequence, a positive subsidy $s > 0$ reduces the minimum level of innovation

[16]We dispense with the discussion of the monetary-policy implications as those rely essentially on the ability of the central bank to rule out a liquidity trap, and therefore the analysis of Benigno and Fornaro (2018) applies to our economy as well.

[17]This obviously does not change the resource constraint because

$$P_t C_t = W_t L_t - T_t + \underbrace{P_t Y_t - W_t L_t - \int_0^{N_t} P_t\left(j\right) y_t\left(j\right) dj}_{\text{Final good firm profits}} +$$
$$+ \underbrace{\int_0^{N_{t-1}} \left(P_t\left(j\right) y_t\left(j\right) - P_t y_t\left(j\right) - R_t\left(j\right) P_t\right)}_{\text{Int. good, incremental innovation profits}} + \underbrace{\int_{N_{t-1}}^{N_t} \left(P_t\left(j\right) y_t\left(j\right) - P_t y_t\left(j\right) - (1-s) R_t\left(j\right) P_t\right)}_{\text{Int. good, radical innovation profits}}$$

and $T_t = s R_t P_t \left(N_t - N_{t-1}\right)$.

intensity along the horizontal margin that is needed to profitably create a new variety, which is now equal to

$$h(\underline{x}_t) = \left(1 - s\frac{\nu}{1+\nu}\right)(\delta\psi L_t)^{\nu}. \tag{4.2}$$

In turn, therefore, the minimum level of technical and entrepreneurial skills is also lower for higher susbisidies, and lower than in the baseline model:

$$\underline{x}(L_t, s) = \left(1 - s\frac{\nu}{1+\nu}\right)^{\varpi} \underline{x}(L_t), \tag{4.3}$$

where $\underline{x}(L_t)$ is the threshold in the baseline model, defined by equation (3.15).

Thus, subsidising radical innovation makes it easier for prospective entrepreneurs to create new varieties because it tilts the relative returns from radical versus incremental innovation in favor of the former. From the perspective of the overall growth process, we can state the following proposition.

**Proposition 6.** *Suppose the assumptions of Propositions 4 and 5 are satisfied.*

*Then the introduction of a positive constant subsidy $s \in (0, 1)$ on the creation of new varieties implies an upward shift and a clockwise rotation of the GG schedule.*

**Proof.** Please refer to Appendix A.5. ■

As we show in Figure 4, the shift and clockwise rotation of the $GG$ schedule push the two unemployment equilibria toward each other and, if the subsidy is large enough, is able to rule out the existence of such unemployment equilibria altogether. Note that the shift of the $GG$ is associated with a shift in the upward-sloping part of aggregate demand as well, because the introduction of the subsidy raises the natural rate of interest consistent with full employment, $i^F$, and therefore the central bank has to adjust the target for its policy rate $\bar{\imath}$ to support the new equilibrium. This in turn implies that the domain of employment over which the zero bound on the interest rate binds becomes smaller, as the central bank has more room to cut its policy rate.

On the one hand, the upward-shift part of Proposition 6 suggests that the introduction of a subsidy allows the economy to reach a new full employment steady state characterised by a higher productivity growth, as it improves the creation of new varieties for any level of employment, and reinforces the growth process induced by vertical innovation also at full employment.

On the other hand, the rotation part of the proposition implies that the introduction of the subsidy makes the $GG$ steeper before its minimum and flatter after it. In other words, the subsidy strengthens the relative weight of the horizontal margin of innovation

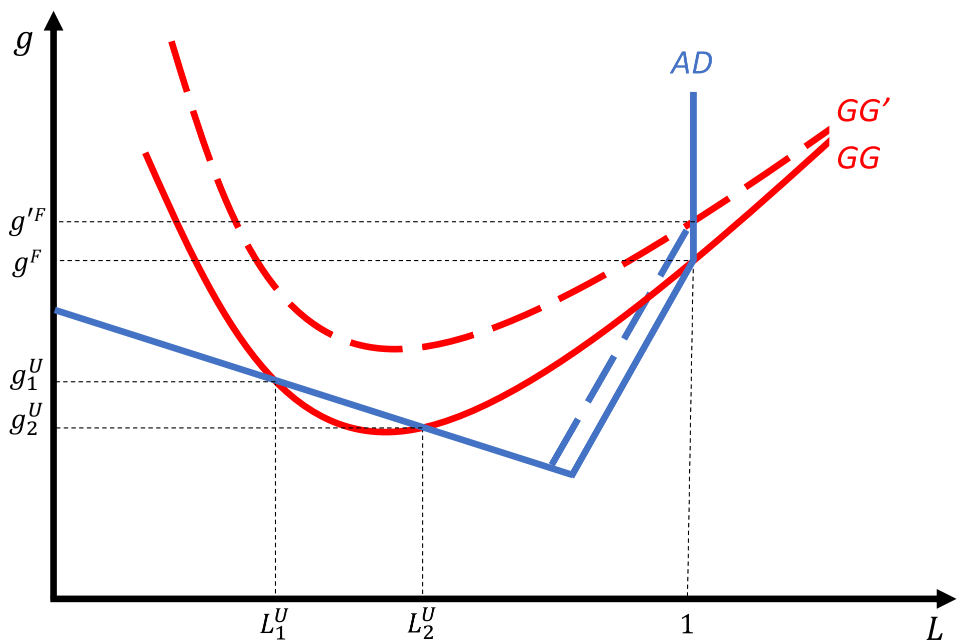


**Figure 4:** Existence and multiplicity of stagnation traps: the role of growth policy. Solid red line: U-shaped growth equation in the absence of subsidies ($s = 0$). Dashed lines: U-shaped growth equation and aggregate demand when radical innovation is subsidised at the constant rate $s > 0$.

over the vertical one on the entire domain of $L_t \in [0, 1]$, but particularly on low values of $L_t$. This is because the opportunity cost argument motivating the pursuit of radical innovation kicks in earlier, being exploration research activities subsidised.

It is important at this point to focus on the differences between our framework with heterogeneous innovation and the seminal paper Benigno and Fornaro (2018), in the behavior under a subsidy policy. Benigno and Fornaro (2018) introduce an additive subsidy that is contingent on growth (or employment) in order to induce the strict convexity (and non-monotonicity, in the case of wage Phillips curve) in the *GG* schedule that is needed in order to rule out the stagnation trap for any relevant value of employment. While both features are powerful and theoretically sensible in delivering the result that a stagnation trap can be ruled out, translating the theoretical result in a practical policy action may be hard. Our framework delivers a much simpler policy prescription. In our economy, the extensive margin of innovation naturally delivers a strictly convex and non-monotonic growth schedule, without the need for a state-contingent subsidy to induce it. As a result, a non-contingent subsidy is able to rule out the (multiple) stagnation traps, by affecting the position of the *GG* locus with a simple tool.

# 5 Conclusion

We have provided empirical evidence that favors a non-monotonic, U-shaped relationship between productivity growth and employment over the period 1948-2019, which reconciles the ”market size” and ”opportunity cost” theories of innovation.

We have introduced heterogeneous innovation—radical, driven by opportunity cost and incremental, driven by market size—in a Keynesian endogenous growth framework with nominal rigidities and monetary policy. The interaction between radical and incremental innovation changes over the business cycle, yielding a U-shaped growth equation. We showed the conditions for existence, determinacy and multiplicity of unemployment equilibria. We found that a U-shaped growth equation can generate stable equilibria with low-growth and low-employment and a liquidity trap. We showed that economic policies aimed at incentivising exploratory research activities can both prevent the exposure to unemployment equilibria and help the economy escape them. This paper shows that a non-contingent subsidy to research exploration activities achieves this.

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# A Proofs of propositions

## A.1 Proposition 2

**Proof.** Let us first prove existence. Setting $L^F = 1$ in the system (3.29)-(3.34) and using (3.31) implies

$$g^F = \Big(1 + \gamma(\delta\psi)^\nu\Big)\Big(1 - \Delta + n(1)\Big) - 1$$

which is positive because of assumption (3.37). Equation (3.29) then implies

$$i^F = \left[\Big(1 + \gamma(\delta\psi)^\nu\Big)\Big(1 - \Delta + n(1)\Big)\right]^{\sigma-1} \frac{\bar{\pi}^w}{\rho\beta} - 1$$

which is positive because of assumption (3.39), and equation (3.34) implies $\bar{\imath} = i^F$ so that the central bank supports the full employment steady state setting the policy rate consistent with it. Using (3.30) one gets

$$c^F = \Psi - \frac{\nu\delta^\nu}{1+\nu}\psi^{1+\nu} > 0$$

where the inequality is ensured by assumption (3.38), given the definitions $\psi \equiv (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}$ and $\Psi \equiv \alpha^{\frac{2\alpha}{1-\alpha}}\left(1-\alpha^2\right)$, and $\alpha \in (0,1)$. Thus a full employment steady state exists. It is also unique because, given $L^F = 1$ equation (3.31) implies only one value of $g^F$ consistent with it and so does equation (3.29) given $g^F$.

Concerning determinacy, note that a log-linear approximation of system (3.29)-(3.34) around the full employment steady state yields:

$$\hat{c}_t = \Omega \hat{L}_t \tag{A.1}$$

$$\sigma \hat{c}_t = -\hat{i}_t + \sigma \mathbb{E}_t \hat{c}_{t+1} - (1-\sigma)\,\mathbb{E}_t \hat{g}_{t+1} \tag{A.2}$$

$$\hat{g}_t = \Phi \hat{L}_t \tag{A.3}$$

$$\hat{i}_t = \phi \hat{L}_t \tag{A.4}$$

where $\Omega \equiv \frac{\frac{1+\alpha}{\alpha} - \nu(\delta\psi)^\nu}{\frac{1+\alpha}{\alpha} - \frac{\nu}{1+\nu}(\delta\psi)^\nu}$, $\Phi \equiv \left[\frac{\gamma\nu(\delta\psi)^\nu}{1+\gamma(\delta\psi)^\nu} + \frac{n'(1)}{1-\Delta+n(1)}\right]$, and we use the notation: $\hat{c}_t \equiv \frac{c_t - c^F}{c^F}$, $\hat{L}_t \equiv \frac{L_t - L^F}{L^F}$, $\hat{i}_t \equiv \frac{i_t - \bar{\imath}}{1+\bar{\imath}}$ and $\hat{g}_t \equiv \frac{g_t - g^F}{1+g^F}$.

Now use (A.1), (A.3) and (A.4) in (A.2) to reduce the system above into the following stochastic difference equation in employment:

$$\hat{L}_t = \frac{\sigma\Omega + (\sigma-1)\Phi}{\sigma\Omega + \phi} \mathbb{E}_t \hat{L}_{t+1}. \tag{A.5}$$

The above equation has a (locally) unique rational-expectations solution, i.e. $\hat{L}_t = 0$, as long as $\left|\frac{\sigma\Omega + (\sigma-1)\Phi}{\sigma\Omega + \phi}\right| < 1$, requiring

$$\phi > (\sigma - 1)\,\Phi$$

which is guaranteed by assumption (3.41). Hence the full employment steady state is locally determinate. ■

## A.2 Proposition 3

**Proof.** We start by proving existence. Setting $i = 0$ in (3.29) we get

$$1 + g^U = \left(\frac{\rho\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} > 1$$

where the inequality is ensured by assumption (3.44). By assumption (3.45), the *GG* starts above the (positive) growth rate $g^U$ and by assumption (3.46) it reaches a minimum below it. This implies that the $GG$ schedule must be downward-sloping at least over some interval $(\bar{L}, L_*)$, for some $\bar{L} \geq 0$ and where $L_* = \arg\min \Big(1 + \gamma(\delta\psi L)^\nu\Big)\Big(1 - \Delta + n(L)\Big)$. Moreover, since $1 + g^A$ is monotonically increasing and convex, since $1 + g^N$ is monotonically decreasing and since, by condition (3.40), $(1 + g^A)(1 + g^N)$ is upward-sloping when $L = 1$, it follows that $(1 + g^A)(1 + g^N)$ is monotonically increasing in the interval $(L_*, 1]$. Finally, given (3.42), and by continuity of (3.31), it follows that the $GG$ crosses the $1 + g^U$ line exactly twice: at $L_1^U$, whereby the $GG$ is downward-sloping, and at $L_2^U$, whereby it is upward-sloping. This implies that there exist at most two unemployment steady states in the interval $(\bar{L}, 1)$ where the growth rate is $g^U < g^F$. If assumption (3.46) holds with equality, the two steady-states overlap, as the $GG$ is tangent to the $1 + g^U$ line at $L_*$ and therefore we have $L_1^U = L_2^U = L_*$ and a flat $GG$ schedule around that steady state.

To show that, in these steady states, $0 < c_1^U \leq c_2^U < c^F$, first note that equilibrium consumption is positive for any level of employment. Indeed, we can write

$$c(L) = \psi L \left[\frac{\Psi}{\psi} - \frac{\nu}{1 + \nu} (\delta\psi L)^\nu\right] \tag{A.6}$$

which is positive as long as $L^\nu < \frac{1+\nu}{\nu}\frac{\Psi}{\psi}(\delta\psi)^{-\nu}$. Moreover, the definitions of $\Psi$ and $\psi$ imply $\frac{\Psi}{\psi} = \frac{1+\alpha}{\alpha} > 1$, from which it follows that the inequality above—and thereby $c(L) > 0$—is always satisfied for $L \in [0, 1]$ under condition (3.38). Finally, note that condition (3.43) implies that equilibrium steady-state consumption is an increasing function of steady-state employment. Indeed, we can write the first derivative of (A.6) as:

$$c'(L) = \Psi - \nu\psi (\delta\psi)^\nu L^\nu \tag{A.7}$$

which is positive if $L^\nu < \frac{1}{\nu}\frac{\Psi}{\psi}(\delta\psi)^{-\nu} = \frac{1+\alpha}{\alpha\nu}(\delta\psi)^{-\nu}$, where the equality follows from the definitions of $\Psi$ and $\psi$. Now, the inequality above—and thereby $c'(L) > 0$—is always satisfied for $L \in [0, 1]$ under conditions (3.38) and (3.43), proving $0 < c_1^U \leq c_2^U < c^F$.

Concerning determinacy, note that a log-linear approximation of system (3.29)-(3.34) around a generic unemployment steady state with employment level $L_0 = \{L_1^U, L_2^U\}$, adjusted consumption $c_0 = \{c_1^U, c_2^U\}$, growth rate $1 + g^U$ satisfying (3.42) and a zero-interest rate $i^U = 0$ yields:

$$\hat{c}_{0,t} = \Omega_0 \hat{L}_{0,t} \tag{A.8}$$

$$\sigma\hat{c}_{0,t} = \sigma\mathbb{E}_t\hat{c}_{0,t+1} - (1 - \sigma)\,\mathbb{E}_t\hat{g}_{t+1}^U \tag{A.9}$$

$$\hat{g}_t^U = \Phi_0 \hat{L}_{0,t} \tag{A.10}$$

where $\Omega_0 \equiv \frac{\frac{1+\alpha}{\alpha}-\nu(\delta\psi L_0)^\nu}{\frac{1+\alpha}{\alpha}-\frac{\nu}{1+\nu}(\delta\psi L_0)^\nu}$, $\Phi_0 \equiv \left[\frac{\gamma\nu(\delta\psi L_0)^\nu}{1+\gamma(\delta\psi L_0)^\nu}+\frac{n'(L_0)L_0}{1-\Delta+n(L_0)}\right]$, and we use the notation: $\hat{c}_{0,t} \equiv \frac{c_t-c_0}{c_0}$, $\hat{L}_{0,t} \equiv \frac{L_t-L_0}{L_0}$, and $\hat{g}_t^U \equiv \frac{g_t-g^U}{1+g^U}$.

Now use (A.8) and (A.10) in (A.9) to reduce the system above into the following stochastic difference equation in employment:

$$\hat{L}_{0,t} = \left(1+\frac{(\sigma-1)\Phi_0}{\sigma\Omega_0}\right)\mathbb{E}_t\hat{L}_{0,t+1}. \tag{A.11}$$

For each equilibrium $L_0 = \{L_1^U, L_2^U\}$ around which the approximation is taken, the above equation has a (locally) unique rational-expectations solution, i.e. $\hat{L}_{0,t} = 0$, as long as $\frac{(\sigma-1)\Phi_0}{\sigma\Omega_0} < 0$, which can only occur if either *i*) $\Omega_0$ and $\Phi_0$ have the same sign and $\sigma < 1$ or *ii*) $\Omega_0$ and $\Phi_0$ have opposite signs and $\sigma > 1$. Given our assumption about $\sigma > 1$, and the fact that assumptions (3.38) and (3.43) imply $\Omega_0 > 0$ for any $L_0 \in [0,1]$, the rational-expectations equilibrium is determinate if and only if

$$\Phi_0 < 0. \tag{A.12}$$

Finally recall that $\Phi_0$ captures the local slope of the $GG$ schedule around $L = L_0$, which is negative (i.e. $\Phi_0 < 0$) around $L_0 = L_1^U$ and positive (i.e. $\Phi_0 > 0$) around $L_0 = L_2^U$, as already shown. This proves that the stagnation trap associated with $L_1^U$ is locally determinate, while the one associated with $L_2^U$ is not. As to the case of a single unemployment steady state associated with $L_0 = L_*$, since in that case the $GG$ schedule is flat around that equilibrium, it follows that $\Phi_0 = 0$ when $L_0 = L_*$, thereby violating condition (A.12). This proves that in this case the stagnation trap is locally indeterminate.

■

## A.3 Proposition 4

**Proof.** To prove determinacy, note that a log-linear approximation of the system around the full employment steady state yields:[18]

$$\hat{c}_t = \Omega \hat{L}_t \tag{A.13}$$

$$\sigma \hat{c}_t = -\hat{i}_t + \sigma \mathbb{E}_t \hat{c}_{t+1} + (\sigma - 1)\, \mathbb{E}_t \hat{g}_{t+1} + \mathbb{E}_t \hat{\pi}^w_{t+1} \tag{A.14}$$

$$\hat{g}_t = \Phi \hat{L}_t \tag{A.15}$$

$$\hat{i}_t = \phi \hat{\pi}^w_t \tag{A.16}$$

$$\hat{\pi}^w_t = \vartheta \hat{L}_t \tag{A.17}$$

where $\Omega \equiv \frac{\frac{1+\alpha}{\alpha} - \nu(\delta\psi)^\nu}{\frac{1+\alpha}{\alpha} - \frac{\nu}{1+\nu}(\delta\psi)^\nu}$, $\Phi \equiv \left[\frac{\gamma\nu(\delta\psi)^\nu}{1+\gamma(\delta\psi)^\nu} + \frac{n'(1)}{1-\Delta+n(1)}\right]$, and we use the notation: $\hat{c}_t \equiv \frac{c_t - c^F}{c^F}$, $\hat{L}_t \equiv \frac{L_t - L^F}{L^F}$, $\hat{i}_t \equiv \frac{i_t - \bar{\imath}}{1+\bar{\imath}}$, $\hat{\pi}^w_t \equiv \frac{\pi^w_t - \pi^*}{\pi^*}$ and $\hat{g}_t \equiv \frac{g_t - g^F}{1+g^F}$.

Now use equations (A.13), (A.15), (A.16) and (A.17) in (A.14) to reduce the system above into the following stochastic difference equation in employment:

$$\hat{L}_t = \frac{\sigma\Omega + (\sigma-1)\Phi + \vartheta}{\sigma\Omega + \phi\vartheta} \mathbb{E}_t \hat{L}_{t+1}. \tag{A.18}$$

The above equation has a (locally) unique rational-expectations solution, i.e. $\hat{L}_t = 0$, as long as $\left|\frac{\sigma\Omega+(\sigma-1)\Phi+\vartheta}{\sigma\Omega+\phi\vartheta}\right| < 1$, requiring

$$\vartheta(\phi - 1) > (\sigma - 1)\, \Phi \tag{A.19}$$

which is guaranteed by assumption (3.51). Hence the full employment steady state is locally determinate.

Finally note that using equation (3.52) in equation (3.50) implies that the local slope of the *AD* schedule from the left of the full-employment steady state is $\vartheta(\phi-1)/(\sigma-1)$. Moreover, recall that $\Phi$ captures the local slope of the *GG* schedule around $L = 1$. This implies that condition (A.19) requires the *AD* schedule to be steeper than the *GG*, around the full-employment steady state, where both schedules are upward sloping. ■

[18] More specifically, we are taking a first-order approximation from the left of the full-employment steady state, as the function becomes vertical at $L = 1$.

## A.4 Proposition 5

**Proof.** To prove determinacy, note that a log-linear approximation of the system around a generic unemployment steady state with employment level $L_0 = \{L_1^U, L_2^U\}$, adjusted consumption $c_0 = \{c_1^U, c_2^U\}$, growth rate $g_0 = \{g_1^U, g_2^U\}$ and interest rate $i^U = 0$ yields:

$$\hat{c}_{0,t} = \Omega_0 \hat{L}_{0,t} \tag{A.20}$$

$$\sigma \hat{c}_{0,t} = \sigma \mathbb{E}_t \hat{c}_{0,t+1} + (\sigma - 1)\, \mathbb{E}_t \hat{g}_{0,t+1} + E_t \hat{\pi}^w_{0,t+1} \tag{A.21}$$

$$\hat{g}_{0,t} = \Phi_0 \hat{L}_{0,t} \tag{A.22}$$

$$\hat{\pi}^w_{0,t} = \vartheta \hat{L}_{0,t} \tag{A.23}$$

where $\Omega_0 \equiv \frac{\frac{1+\alpha}{\alpha} - \nu(\delta\psi L_0)^\nu}{\frac{1+\alpha}{\alpha} - \frac{\nu}{1+\nu}(\delta\psi L_0)^\nu}$, $\Phi_0 \equiv \left[\frac{\gamma\nu(\delta\psi L_0)^\nu}{1+\gamma(\delta\psi L_0)^\nu} + \frac{n'(L_0)L_0}{1-\Delta+n(L_0)}\right]$, and we use the notation: $\hat{c}_{0,t} \equiv \frac{c_t - c_0}{c_0}$, $\hat{L}_{0,t} \equiv \frac{L_t - L_0}{L_0}$, $\hat{\pi}^w_{0,t} \equiv \frac{\pi^w_t - \pi^w_0}{\pi^w_0}$, and $\hat{g}_{0,t} \equiv \frac{g_t - g_0}{1+g_0}$.

Now use (A.20), (A.22) and (A.23) in (A.21) to reduce the system above into the following stochastic difference equation in employment:

$$\hat{L}_{0,t} = \left(1 + \frac{(\sigma-1)\Phi_0 + \vartheta}{\sigma\Omega_0}\right) \mathbb{E}_t \hat{L}_{0,t+1}. \tag{A.24}$$

For each equilibrium $L_0 = \{L_1^U, L_2^U\}$ around which the approximation is taken, the above equation has a (locally) unique rational-expectations solution, i.e. $\hat{L}_{0,t} = 0$, as long as $\frac{(\sigma-1)\Phi_0+\vartheta}{\sigma\Omega_0} < 0$. Since assumptions (3.38) and (3.43) imply $\Omega_0 > 0$ for any $L_0 \in [0,1]$, the equilibrium is locally determinate if and only if $(\sigma-1)\Phi_0 + \vartheta < 0$, i.e.:

$$\Phi_0 < \frac{-\vartheta}{\sigma - 1} < 0. \tag{A.25}$$

Now note that using (3.52) in (3.50) implies that the local slope of the $AD$ schedule around a generic reference point $L = L_0$ in a liquidity trap is $-\vartheta/(\sigma-1) < 0$. Moreover, recall that $\Phi_0$ captures the local slope of the $GG$ schedule around $L = L_0$. Therefore, equation (A.25) implies that local determinacy requires indeed the slope of the $AD$ schedule to be larger than the slope of the $GG$. This makes the rest of the proof straightforward, given continuity of both the $GG$ and $AD$ schedules.

If only one steady state exists, it must be that the $AD$ schedule is tangent to the $GG$, in which case the two slopes are equal and they are both negative:

$$\Phi_0 = \frac{-\vartheta}{\sigma - 1} < 0.$$

This violates condition (A.25), proving that in this case the unemployment steady state is locally indeterminate.

If two steady states exist, on the other hand, it must be that the $AD$ intersects the $GG$ in two separate points. In this case, on the one hand, the intersection associated with the lower employment $L_0 = L_1^U$ must be characterized by a steeper $GG$ relative to the $AD$, where both are downward sloping:

$$\Phi_0 < \frac{-\vartheta}{\sigma - 1} < 0.$$

This means that this equilibrium satisfies condition (A.25), and it is therefore determinate. On the other hand, at the intersection associated with the higher employment $L_0 = L_2^U$, the $GG$ must be either downward sloping and flatter than the $AD$, i.e.

$$\frac{-\vartheta}{\sigma - 1} < \Phi_0 < 0, \tag{A.26}$$

or upward sloping, i.e.

$$\frac{-\vartheta}{\sigma - 1} < 0 < \Phi_0.$$

In either case, this implies that condition (A.25) is violated, and the equilibrium is therefore indeterminate. It is worth emphasizing that (A.26) implies that in order for the stagnation trap to be indeterminate, it does not need to be associated with the upward-sloping part of the $GG$, unlike in the simple model. ■

## A.5 Proposition 6

**Proof.** To prove the upward shift of the $GG$ schedule, we show that the mass of innovators participating in the creation of new varieties is increasing in the subsidy $s$ at the full-employment steady-state. Recall first that the mass of horizontal innovators is defined as:

$$n_t(L_t, s) = \int_{\underline{x}(L_t,s)}^{1} h(x) f(x) dx,$$

where we are using $h(x) = \kappa x^{\frac{1}{\omega}}$ and equation (4.3), and where $f(x)$ is a unimodal and right-skewed beta density function.

Then, compute the derivative of $n_t$ with respect to $s$, and evaluate it at $L = 1$. Using the Leibniz integral rule one obtains:

$$\frac{\partial n_t}{\partial s} = -h(\underline{x}_t) f(\underline{x}_t) \frac{\partial \underline{x}_t}{\partial s}. \tag{A.27}$$

Note now that the derivative of the threshold, evaluated at full-employment, is:

$$\frac{\partial \underline{x}_t}{\partial s} = -\varpi \left(1 - s\frac{\nu}{1+\nu}\right)^{\varpi-1} \frac{\nu}{1+\nu}\kappa^{-\varpi} (\delta\psi)^{\nu\varpi} < 0. \tag{A.28}$$

From using the above and $h(\underline{x}(1, s)) = \left(1 - s\frac{\nu}{1+\nu}\right) (\delta\psi)^{\nu}$ into (A.27), it follows that:

$$\frac{\partial n_t}{\partial s} = f(\underline{x}(1, s))\varpi \left(1 - s\frac{\nu}{1+\nu}\right)^{\varpi} \frac{\nu}{1+\nu}\kappa^{-\varpi} (\delta\psi)^{\nu(1+\varpi)} > 0.$$

Thus, an increase in the subsidy raises the mass of successful horizontal innovators and thereby the mass of newly-created varieties.

To prove the clockwise rotation, we show that the *GG* schedule flattens around the full-employment steady state. Compute the second cross-derivative of $n_t$ with respect to $L_t$ and $s$:

$$\frac{\partial^2 n_t}{\partial L_t \partial s} = -\left[h'(\underline{x}_t)\frac{\partial \underline{x}_t}{\partial s} f(\underline{x}_t)\frac{\partial \underline{x}_t}{\partial L_t} + h(\underline{x}_t)f'(\underline{x}_t)\frac{\partial \underline{x}_t}{\partial s}\frac{\partial \underline{x}_t}{\partial L_t} + h(\underline{x}_t)f(\underline{x}_t)\frac{\partial^2 \underline{x}_t}{\partial L_t \partial s}\right]. \tag{A.29}$$

To evaluate the sign of the cross derivative, note the following. $f(x) > 0$ by definition; $f'(x) < 0$ and arbitrarily close to 0 for $x$ sufficiently close to 1 and/or the distribution $f(x)$ sufficiently right-skewed; $\frac{\partial \underline{x}}{\partial s} < 0$ as trivially implied by equation (4.3); $\frac{\partial \underline{x}}{\partial L_t} > 0$ as shown by (3.18); $h(x) > 0$ and $h'(x) > 0$, by assumption; $\frac{\partial^2 \underline{x}}{\partial L_t \partial s} < 0$, which is trivially implied by equation (3.18), modified to account for the subsidy as follows:

$$\frac{\partial \underline{x}(L_t, s)}{\partial L_t} = \nu\varpi\kappa^{-\varpi}\frac{(\delta\psi L_t)^{\nu\varpi}}{L_t}\left(1 - s\frac{\nu}{1+\nu}\right)^{\varpi}. \tag{A.30}$$

The above implies that the first term in (A.29) is negative, the second is positive, and the third is negative. Now note: *i*) the positive term is dominated by the two negative ones for values of the threshold $\underline{x}_t$ sufficiently high, which drives the magnitude of $f'(x)$ towards zero and *ii*) around the full-employment equilibrium the value of $\underline{x}_t$ reaches its maximum value. As a consequence, the slope of the *GG* schedule around the full-employment steady state decreases as the subsidy increases, as long as the distribution $f(x)$ is sufficiently right-skewed. We check in the calibrated version of our model that this is indeed the case. ■

# B Data Appendix

## B.1 Data description

Most of our variables can be downloaded from the FRED database. Our sample contains quarterly observations on labour productivity growth, employment, population (CNP16OV), R&D (Y006RC1Q027SBEA), liabilities of the nonfinancial corporate business sector, consumption (DPCERA3M086SBEA), CPI inflation (CPIAUCSL), PCE inflation (PCEPI) and imports price deflator (A021RD3Q086SBEA), over the period 1984Q1-2019Q4[19]. Nominal variables are deflated with GDP deflator (GDPDEF).

The population series is the Civilian Non-institutional Population from the BLS, where high-frequency variation has been removed with the Hodrick and Prescott (1997)'s filter with a smoothing parameter lambda of 1600, in order to avoid spikes due to census to affect the series.

Regarding the employment rate, we take the complement to one of the unemployment rate (UNRATE), quarterly and seasonally adjusted, from the FRED database. We scale it by its maximum so as to get a number between 0 and 1 whose maximum is 1, as in our model specification.

Our preferred measure of labour productivity is nonfarm business sector labor productivity (PRS85006091) from the BLS, available on the FRED database at a quarterly frequency.

As a robustness, we employ two additional measures of productivity growth: labour productivity of the business sector and total factor productivity, both from Fernald (2014).

[19] Variables' identifiers in parenthesis.

# C Additional Tables and Figures

**Table 5:** The U-shaped Growth Equation: estimation results using Fernald (2014) measure of labor productivity

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | L. 1984–2019 | L. 1984–2000 | L. 1984–2007 | L. 2000–2019 | PWL 1984–2019 | Q. 1984–2019 |
| Empl | -0.025 | 0.069 | 0.064 | -0.042 | -0.264* | -7.513** |
| | (0.030) | (0.059) | (0.050) | (0.036) | (0.146) | (3.135) |
| Empl*Threshold | | | | | 0.340** | |
| | | | | | (0.156) | |
| Empl2 | | | | | | 3.901** |
| | | | | | | (1.633) |
| R2 | 0.100 | 0.265 | 0.163 | 0.053 | 0.155 | 0.136 |
| Controls | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 144 | 65 | 96 | 80 | 144 | 144 |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. "L." stands for linear regression specified in Model 2.2. "PWL" stands for piecewise linear regression specified in Model 2.4. "Q." stands for quadratic regression specified in Model 2.5.

**Table 6:** The U-shaped Growth Equation: estimation results using Fernald (2014) measure of total factor productivity

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | L. 1984–2019 | L. 1984–2000 | L. 1984–2007 | L. 2000–2019 | PWL 1984–2019 | Q. 1984–2019 |
| Empl | -0.056* | 0.013 | -0.001 | -0.080** | -0.243*** | -6.411** |
| | (0.030) | (0.062) | (0.053) | (0.034) | (0.085) | (3.101) |
| Empl*Threshold | | | | | 0.155 | |
| | | | | | (0.113) | |
| Empl2 | | | | | | 3.311** |
| | | | | | | (1.616) |
| R2 | 0.225 | 0.275 | 0.179 | 0.249 | 0.302 | 0.248 |
| Controls | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 144 | 65 | 96 | 80 | 144 | 144 |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. "L." stands for linear regression specified in Model 2.2. "PWL" stands for piecewise linear regression specified in Model 2.4. "Q." stands for quadratic regression specified in Model 2.5.

**Table 7:** The U-shaped Growth Equation: threshold regressions with different dependent variables

| | Fernald (2014) Labor productivity | | Fernald (2014) TFP | |
|---|---|---|---|---|
| | Region 1<br>$L_t \leq .953$ | Region 2<br>$L_t > .953$ | Region 1<br>$L_t \leq .961$ | Region 2<br>$L_t > .961$ |
| $L_t$ | -.362***<br>(.141) | .049<br>(.0556) | -.243**<br>(.085) | -.088<br>(.075) |
| BIC | -1470.451 | | -195.49 | |
| HQIC | -1484.552 | | -229.61 | |
| Controls | ✓ | | ✓ | |
| $N$ | 144 | | 144 | |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. The thresholds are estimated minimizing the Bayesian information criteria.

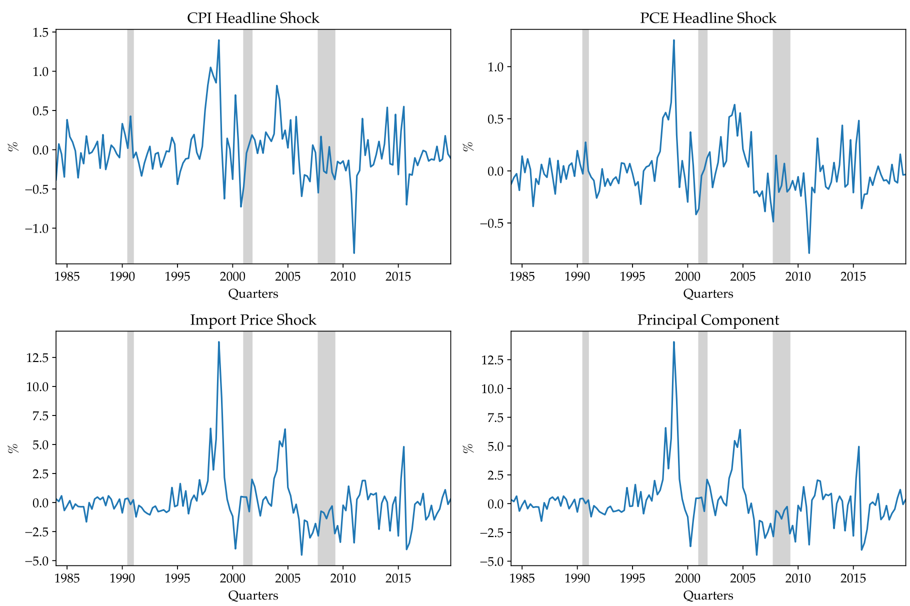


**Figure 5:** The figure plots the three measures of supply shocks, together with their first principal component. Time period is 1984Q1-2019Q4.

**Table 8:** The U-shaped Growth Equation: estimation results adding more controls

| | (1)<br>L. 1984–2019 | (2)<br>L. 1984–2000 | (3)<br>L. 1984–2007 | (4)<br>L. 2000–2019 | (5)<br>Q. 1984–2019 |
|---|---|---|---|---|---|
| Empl | 0.005 | -0.110*** | -0.088*** | 0.031 | -7.620*** |
| | (0.017) | (0.040) | (0.032) | (0.020) | (1.692) |
| Empl*Threshold | | | | | |
| | | | | | |
| Empl2 | | | | | 3.971*** |
| | | | | | (0.881) |
| R2 | 0.300 | 0.404 | 0.421 | 0.448 | 0.391 |
| Controls | Yes | Yes | Yes | Yes | Yes |
| N | 144 | 65 | 96 | 80 | 144 |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. "L." stands for linear regression specified in Model 2.2. "Q." stands for quadratic regression specified in Model 2.5. The vector of controls now includes, in addition to the baseline case, also population growth, growth rate of liabilities of the nonfinancial corporate business sector and the growth rate of investment in R&D.

**Table 9:** The U-shaped Growth Equation: estimation results with the difference between headline and core CPI inflation as a proxy for the supply shock

| | (1)<br>L. 1984–2019 | (2)<br>L. 1984–2000 | (3)<br>L. 1984–2007 | (4)<br>L. 2000–2019 | (5)<br>PWL 1984–2019 | (6)<br>Q. 1984–2019 |
|---|---|---|---|---|---|---|
| Empl | -0.013 | 0.037 | 0.028 | -0.021 | -0.321*** | -6.000*** |
| | (0.019) | (0.038) | (0.035) | (0.023) | (0.095) | (1.926) |
| Empl*Threshold | | | | | 0.372*** | |
| | | | | | (0.100) | |
| Empl2 | | | | | | 3.120*** |
| | | | | | | (1.003) |
| R2 | 0.052 | 0.135 | 0.028 | 0.055 | 0.186 | 0.114 |
| Controls | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 144 | 65 | 96 | 80 | 144 | 144 |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. "L." stands for linear regression specified in Model 2.2. "PWL" stands for piecewise linear regression specified in Model 2.4. "Q." stands for quadratic regression specified in Model 2.5.

**Table 10:** The U-shaped Growth Equation: estimation results with the difference between headline and PCE price index inflation as a proxy for the supply shock

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | L. 1984–2019 | L. 1984–2000 | L. 1984–2007 | L. 2000–2019 | PWL 1984–2019 | Q. 1984–2019 |
| Empl | -0.013 | 0.031 | 0.027 | -0.021 | -0.307*** | -5.988*** |
| | (0.019) | (0.038) | (0.035) | (0.023) | (0.091) | (1.919) |
| Empl*Threshold | | | | | 0.359*** | |
| | | | | | (0.096) | |
| Empl2 | | | | | | 3.114*** |
| | | | | | | (1.000) |
| R2 | 0.052 | 0.118 | 0.027 | 0.054 | 0.188 | 0.114 |
| Controls | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 144 | 65 | 96 | 80 | 144 | 144 |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. "L." stands for linear regression specified in Model 2.2. "PWL" stands for piecewise linear regression specified in Model 2.4. "Q." stands for quadratic regression specified in Model 2.5.

**Table 11:** The U-shaped Growth Equation: estimation results with he difference between the change in the import prices and the GDP deflator as a proxy for the supply shock

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | L. 1984–2019 | L. 1984–2000 | L. 1984–2007 | L. 2000–2019 | PWL 1984–2019 | Q. 1984–2019 |
| Empl | -0.013 | 0.016 | 0.019 | -0.018 | -0.292*** | -5.774*** |
| | (0.019) | (0.036) | (0.033) | (0.023) | (0.098) | (1.915) |
| Empl*Threshold | | | | | 0.343*** | |
| | | | | | (0.102) | |
| Empl2 | | | | | | 3.002*** |
| | | | | | | (0.997) |
| R2 | 0.053 | 0.097 | 0.021 | 0.059 | 0.170 | 0.111 |
| Controls | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 144 | 65 | 96 | 80 | 144 | 144 |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. "L." stands for linear regression specified in Model 2.2. "PWL" stands for piecewise linear regression specified in Model 2.4. "Q." stands for quadratic regression specified in Model 2.5.

**Table 12:** The U-shaped Growth Equation: threshold regression with lagged employment

| | Employment | |
|---|---|---|
| | Region 1 $L_{t-1} \leq .953$ | Region 2 $L_{t-1} > .953$ |
| $L_{t-1}$ | -.112 | .051 |
| | (.076) | (.036) |
| BIC | -1608.36 | |
| HQIC | -1622.47 | |
| Controls | ✓ | |
| $N$ | 144 | |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. The threshold is estimated minimizing the Bayesian information criteria.

**Table 13:** The U-shaped Growth Equation: estimation results with lagged employment

| | (1) L. 1984–2019 | (2) L. 1984–2000 | (3) L. 1984–2007 | (4) L. 2000–2019 | (5) PWL 1984–2019 | (6) Q. 1984–2019 |
|---|---|---|---|---|---|---|
| Lagged Empl | -0.001 | 0.026 | 0.028 | -0.007 | -0.112 | -4.860** |
| | (0.019) | (0.036) | (0.032) | (0.023) | (0.076) | (1.979) |
| Lagged-Empl*Threshold | | | | | 0.158* | |
| | | | | | (0.084) | |
| Lagged Empl2 | | | | | | 2.532** |
| | | | | | | (1.031) |
| R2 | 0.050 | 0.103 | 0.026 | 0.052 | 0.121 | 0.090 |
| Controls | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 144 | 65 | 96 | 80 | 144 | 144 |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. "L." stands for linear regression specified in Model 2.2. "PWL" stands for piecewise linear regression specified in Model 2.4. "Q." stands for quadratic regression specified in Model 2.5.

**Table 14:** The U-shaped Growth Equation: threshold regression with two-quarters lagged employment

| | Employment | |
|---|---|---|
| | Region 1 $L_{t-2} \leq .976$ | Region 2 $L_{t-2} > .976$ |
| $L_{t-2}$ | -.020 | .087 |
| | (.024) | (.171) |
| BIC | -1606.11 | |
| HQIC | -1620.12 | |
| Controls | ✓ | |
| $N$ | 144 | |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. The threshold is estimated minimizing the Bayesian information criteria.

**Table 15:** The U-shaped Growth Equation: estimation results with two quarters lagged employment

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | L. 1984–2019 | L. 1984–2000 | L. 1984–2007 | L. 2000–2019 | PWL 1984–2019 | Q. 1984–2019 |
| Lagged Empl | 0.013 | 0.034 | 0.043 | 0.009 | -0.020 | -3.751* |
| | (0.018) | (0.034) | (0.030) | (0.023) | (0.024) | (1.990) |
| Lagged-Empl*Threshold | | | | | -0.066 | |
| | | | | | (0.172) | |
| Lagged Empl2 | | | | | | 1.963* |
| | | | | | | (1.037) |
| R2 | 0.054 | 0.110 | 0.039 | 0.053 | 0.106 | 0.078 |
| Controls | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 144 | 65 | 96 | 80 | 144 | 144 |

**Notes:** Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively. “L.” stands for linear regression specified in Model 2.2. “PWL” stands for piecewise linear regression specified in Model 2.4. “Q.” stands for quadratic regression specified in Model 2.5.